4.1 a) $\begin{cases} 3x + y = -13 \\ -11x - 8y = -13 \end{cases} \begin{vmatrix} \cdot 8 \\ \cdot 1 \end{vmatrix} \Leftrightarrow \begin{cases} (8\cdot 3 + 1(-11))x \overset{(*)}{=} 8(-13) - 13 \\ y = -3x - 13 \end{cases} \Leftrightarrow \begin{cases} 13x = -9\cdot 13 \,|:13 \Rightarrow x = -9 \\ y = -3(-9) - 13 = 27 - 13 = 14 \end{cases}$

on ds 2 a son signe changés

une des 2 eq-ations

donc $S = \{(-9; 14)\}$

(*) 8 fois ligne 1 + 1 ligne 2 ainsi il n'y a plus de y

b) $\begin{cases} -x + y = -20 \\ -3x - y = 4 \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} -4x = -16 \\ y = x - 20 \end{cases} \Leftrightarrow \begin{cases} x = 4 \\ y = 4 - 20 = -16 \end{cases}$ donc $S = \{(4; -16)\}$

c) $\begin{cases} 5x + 2y = -102 \\ 6x - y = -68 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \Leftrightarrow \begin{cases} 17x = -238 \\ 6x + 68 = y \end{cases} \Leftrightarrow \begin{cases} x = -\frac{238}{17} = -14 \\ 6(-14) + 68 = -84 + 68 = -16 = y \end{cases}$ donc $S = \{(-14; -16)\}$

d) $\begin{cases} -5x - y = 20 \\ -5x + 3y = 60 \end{cases} \begin{vmatrix} -1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} 4y = 40 \\ -y - 20 = 5x \end{cases} \Leftrightarrow \begin{cases} y = 10 \\ -10 - 20 = 5x \end{cases} \Leftrightarrow \begin{cases} y = 10 \\ x = \frac{-30}{5} = -6 \end{cases}$ donc $S = \{(-6; 10)\}$

e) $\begin{cases} -10x + 11y = 135 \\ -5x + 10y = -180 \end{cases} \begin{vmatrix} -1 \\ 2 \end{vmatrix} \Leftrightarrow \begin{cases} 9y = -495 \\ 10y + 180 = 5x \end{cases} \Leftrightarrow \begin{cases} y = \frac{-495}{9} = -55 \\ -550 + 180 = 5x \end{cases} \Leftrightarrow \begin{cases} y = -55 \\ x = -\frac{370}{5} = -74 \end{cases}$ $S = \{(-74; -55)\}$

f) $\begin{cases} 2x + y = 10 \\ -3x + y = -25 \end{cases} \begin{vmatrix} 1 \\ -1 \end{vmatrix} \Leftrightarrow \begin{cases} 5x = 35 \\ y = 3x - 25 \end{cases} \Leftrightarrow \begin{cases} x = \frac{35}{5} = 7 \\ y = 21 - 25 = -4 \end{cases}$ donc $S = \{(7; -4)\}$

4.2 a) $\begin{cases} -2x + 2y = 4 \\ -9x - y = -7 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \Leftrightarrow \begin{cases} -20x = -10 \\ 7 - 9x = y \end{cases} \Leftrightarrow \begin{cases} x = \frac{-10}{-20} = \frac{1}{2} \\ y = 7 - \frac{9}{2} = \frac{14-9}{2} = \frac{5}{2} \end{cases}$ donc $S = \{(\frac{1}{2}; \frac{5}{2})\}$

b) $\begin{cases} -x + 4y = -2 \\ x - 6y = -4 \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} -2y = -6 \\ x = 6y - 4 \end{cases} \Leftrightarrow \begin{cases} y = \frac{-6}{-2} = 3 \\ x = 18 - 4 = 14 \end{cases}$ donc $S = \{(14; 3)\}$

c) $\begin{cases} x + y = 3 \\ -x + 5y = 8 \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} 6y = 11 \\ x = 3 - y \end{cases} \Leftrightarrow \begin{cases} y = \frac{11}{6} \\ x = 3 - \frac{11}{6} = \frac{18-11}{6} = \frac{7}{6} \end{cases}$ donc $S = \{(\frac{7}{6}; \frac{11}{6})\}$

d) $\begin{cases} -x + 8y = -1 \\ -x + 3y = -4 \end{cases} \begin{vmatrix} 1 \\ -1 \end{vmatrix} \Leftrightarrow \begin{cases} 5y = 3 \\ 4 + 3y = x \end{cases} \Leftrightarrow \begin{cases} y = \frac{3}{5} \\ x = 4 + \frac{9}{5} = \frac{20+9}{5} \end{cases}$ donc $S = \{(\frac{29}{5}; \frac{3}{5})\}$

e) $\begin{cases} x - y = 1 \\ -5x - 2y = -1 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} 7x = 3 \\ x - 1 = y \end{cases} \Leftrightarrow \begin{cases} x = \frac{3}{7} \\ y = \frac{3}{7} - 1 = \frac{3-7}{7} = \frac{-4}{7} \end{cases}$ donc $S = \{(\frac{3}{7}; -\frac{4}{7})\}$

f) $\begin{cases} x - y = 5 \\ 6x + y = -11 \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} 7x = -6 \\ y = x - 5 \end{cases} \Leftrightarrow \begin{cases} x = -\frac{6}{7} \\ y = -\frac{6}{7} - 5 = \frac{-6-35}{7} = \frac{-41}{7} \end{cases}$ donc $S = \{(-\frac{6}{7}; -\frac{41}{7})\}$

4.3 a) $\begin{cases} -\frac{3}{4}x - \frac{2}{7}y = 5 \,|\cdot 4\cdot 7 \\ -3x - \frac{1}{2}y = 2 \,|\cdot(-2) \end{cases} \Leftrightarrow \begin{cases} -21x - 8y = 140 \\ 6x + y = -4 \end{cases} \begin{vmatrix} 1 \\ 8 \end{vmatrix} \Leftrightarrow \begin{cases} 27x = 108 \\ y = -4 - 6x \end{cases} \Leftrightarrow \begin{cases} x = \frac{108}{27} = 4 \\ y = -4 - 24 = -28 \end{cases}$ donc $S = \{(4; -28)\}$

b) $\begin{cases} \frac{3}{2}x - \frac{2}{3}y = 1 \,|\cdot 2\cdot 3 \\ -\frac{7}{3}x + y = \frac{7}{3} \,|\cdot 3 \end{cases} \Leftrightarrow \begin{cases} 9x - 4y = 6 \\ -7x + 3y = 7 \end{cases} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \Leftrightarrow \begin{cases} -x = 46 \\ 3y = 7(-46) + 7 = -315 \end{cases} \Leftrightarrow \begin{cases} x = -46 \\ y = \frac{-315}{3} = -105 \end{cases}$ donc $S = \{(-46; -105)\}$

c) $\begin{cases} x + y = \frac{7}{5} \\ -3x - 5y = -1 \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} -2y = \frac{21}{5} - 1 = \frac{21-5}{5} = \frac{16}{5} \\ x = \frac{7}{5} - y \end{cases} \Leftrightarrow \begin{cases} y = -\frac{8}{5} \\ x = \frac{7}{5} + \frac{8}{5} = \frac{15}{5} = 3 \end{cases}$ donc $S = \{(3; -\frac{8}{5})\}$

d) $\begin{cases} \frac{1}{3}x - y = 1 \\ x + y = -3 \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix} \Leftrightarrow \begin{cases} \frac{4}{3}x = -2 \\ y = -3 - x \end{cases} \Leftrightarrow \begin{cases} x = -2\cdot\frac{3}{4} = -\frac{3}{2} \\ y = -3 + \frac{3}{2} = \frac{-6+3}{2} = -\frac{3}{2} \end{cases} \Rightarrow S = \{(-\frac{3}{2}; -\frac{3}{2})\}$

e) $\begin{cases} x - \frac{2}{5}y = 5 \\ x - y = \frac{1}{5} \end{cases} \begin{vmatrix} 1 \\ -1 \end{vmatrix} \Leftrightarrow \begin{cases} \frac{1}{5}y = 5 - \frac{1}{5} = \frac{25-1}{5} = \frac{24}{5} \\ x = y - \frac{1}{5} \end{cases} \Leftrightarrow \begin{cases} y = \frac{72}{5} \\ x = \frac{72}{5} - \frac{1}{5} = \frac{71}{5} \end{cases}$ $S = \{(\frac{71}{5}; \frac{72}{5})\}$

f) $\begin{cases} \frac{3}{4}x - \frac{2}{3}y = -2 \,|\cdot 4\cdot 3 \\ 2x + 5y = -\frac{1}{2} \,|\cdot 2 \end{cases} \Leftrightarrow \begin{cases} 9x - 8y = -24 \\ 4x + 10y = -1 \end{cases} \begin{vmatrix} 5 \\ 4 \end{vmatrix} \Leftrightarrow \begin{cases} 61x = -124 \\ 10y = -1 - 4x \end{cases} \Leftrightarrow \begin{cases} x = -\frac{124}{61} \\ 10y = -1 + \frac{496}{61} = \frac{-61+496}{61} = \frac{435}{61} \end{cases}$ donc $S = \{(-\frac{124}{61}; \frac{435}{610})\}$

f') $\begin{cases} \frac{3}{4}x - \frac{2}{3}y = \boxed{-4} \,|\cdot 4\cdot 3 \\ 2x + 5y = -\frac{1}{2} \,|\cdot 2 \end{cases} \Leftrightarrow \begin{cases} 9x - 8y = -48 \\ 4x + 10y = -1 \end{cases} \begin{vmatrix} 5 \\ 4 \end{vmatrix} \Leftrightarrow \begin{cases} 61x = -244 \\ 10y = -1 - 4x \end{cases} \Leftrightarrow \begin{cases} x = \frac{-244}{61} = -4 \\ 10y = -1 + 16 = 15 \end{cases} \Leftrightarrow \begin{cases} x = -4 \\ y = \frac{15}{10} = \frac{3\cdot 5}{2\cdot 5} = \frac{3}{2} \end{cases}$ donc $S = \{(-4; \frac{3}{2})\}$