

2.8

$$1) a^6 - 2a^3 + 1 = (a^3 - 1)^2 = ((a - 1)(a^2 + a + 1))^2 = (a - 1)^2 (a^2 + a + 1)^2$$

$$2) 10xz - 10z - x^2 + x = 10z(x - 1) - x(x - 1) = (x - 1)(10z - x)$$

$$3) 2a - 8a^3 + 8a^5 = 2a(1 - 4a^2 + 4a^4) = 2a(1 - 2a^2)^2$$

$$4) 4x^2y^2 - 20xy + 25 = (2xy - 5)^2$$

$$5) x^2 - 22x + 85 = (x - 5)(x - 17)$$

$$6) x^4y^3 + 3x^3y^2 + 3x^2y + x = x(x^3y^3 + 3x^2y^2 + 3xy + 1) = x(xy + 1)^3$$

$$7) a^8 - 2a^4 + 1 = (a^4 - 1)^2 = ((a^2 - 1)(a^2 + 1))^2 = ((a - 1)(a + 1)(a^2 + 1))^2 = (a - 1)^2 (a + 1)^2 (a^2 + 1)^2$$

$$8) (x^2 + 1)^2 - 4x^2 = ((x^2 + 1) - 2x)((x^2 + 1) + 2x) = (x^2 - 2x + 1)(x^2 + 2x + 1) = (x - 1)^2 (x + 1)^2$$

$$9) 6x^2 + xy + 18xz + 3yz = x(6x + y) + 3z(6x + y) = (6x + y)(x + 3z)$$

$$10) x^6 - 1 = (x^3 - 1)(x^3 + 1) = (x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$$

$$11) (x^2 + 1)^2 - 4x^2 = ((x^2 + 1) - 2x)((x^2 + 1) + 2x) = (x^2 - 2x + 1)(x^2 + 2x + 1) = (x - 1)^2 (x + 1)^2$$

$$12) 18x^4 - 2(x^2 - 1)^2 = 2(9x^4 - (x^2 - 1)^2) = 2((3x^2 - (x^2 - 1))(3x^2 + (x^2 - 1))) = 2((2x^2 + 1)(4x^2 - 1)) = 2(2x^2 + 1)(2x + 1)(2x - 1)$$

$$13) x^4y^3 + 3x^3y^2 + 3x^2y + x = x(x^3y^3 + 3x^2y^2 + 3xy + 1) = x(xy + 1)^3$$

$$14) a^4 + 2a^3 - 2a - 1 = (a^4 - 1) + (2a^3 - 2a) = (a^2 + 1)(a^2 - 1) + 2a(a^2 - 1) = (a^2 - 1)((a^2 + 1) + 2a) = (a^2 - 1)(a^2 + 2a + 1) = (a - 1)(a + 1)(a + 1)^2 = (a - 1)(a + 1)^3$$

$$15) (x^2 - 1)^2 - (x + 1)(x - 1)^3 = (x - 1)^2(x + 1)^2 - (x + 1)(x - 1)^3 = (x - 1)^2(x + 1)((x + 1) - (x - 1)) = 2(x - 1)^2(x + 1)$$

$$16) 4x^2y^2 - (z^2 - y^2 - x^2)^2 = (2xy - (z^2 - y^2 - x^2))(2xy + (z^2 - y^2 - x^2)) = ((x^2 + 2xy + y^2) - z^2)(z^2 - (x^2 - 2xy + y^2)) = ((x + y)^2 - z^2)(z^2 - (x - y)^2) = ((x + y) - z)((x + y) + z)(z - (x - y))(z + (x - y)) = (x + y - z)(x + y + z)(-x + y + z)(x - y + z)$$

$$17) x^2 - 2xy + y^2 - 1 = (x^2 - 2xy + y^2) - 1 = (x - y)^2 - 1 = ((x - y) - 1)((x - y) + 1) = (x - y - 1)(x - y + 1)$$

$$18) \quad x^3 y + 6 x^2 y + 9 x y = x y (x^2 + 6 x + 9) = x y (x + 3)^2$$

$$\begin{aligned} 19) \quad & x(x-2)^2 + (x^3 - 8) - 3(x^2 - 4) = \\ & x(x-2)^2 + (x-2)(x^2 + 2x + 4) - 3(x-2)(x+2) = \\ & (x-2)(x(x-2) + (x^2 + 2x + 4) - 3(x+2)) = \\ & (x-2)(x^2 - 2x + x^2 + 2x + 4 - 3x - 6) = (x-2)(2x^2 - 3x - 2) \end{aligned}$$

Réolvons l'équation $2x^2 - 3x - 2 = 0$.

$$\Delta = (-3)^2 - 4 \cdot 2 \cdot (-2) = 25 = 5^2 > 0$$

$$x_1 = \frac{-(-3)-5}{2 \cdot 2} = -\frac{1}{2} \quad x_2 = \frac{-(-3)+5}{2 \cdot 2} = 2$$

$$2x^2 - 3x - 2 = 2\left(x + \frac{1}{2}\right)(x-2) = (2x+1)(x-2)$$

$$\begin{aligned} \text{On conclut que } & x(x-2)^2 + (x^3 - 8) - 3(x^2 - 4) = (x-2)(2x^2 - 3x - 2) = \\ & (x-2)(2x+1)(x-2) = (x-2)^2(2x+1) \end{aligned}$$

$$\begin{aligned} 20) \quad & 4(a+b)^3 - 4(a-b)^3 = 4((a+b)^3 - (a-b)^3) = \\ & 4((a+b) - (a-b))((a+b)^2 + (a+b)(a-b) + (a-b)^2) = \\ & 4(2b)(a^2 + 2ab + b^2 + a^2 - b^2 + a^2 - 2ab + b^2) = 8b(3a^2 + b^2) \end{aligned}$$

$$21) \quad x^3 y + 7 x^2 y + 6 x y = x y (x^2 + 7 x + 6) = x y (x + 1)(x + 6)$$

$$\begin{aligned} 22) \quad & x^4 y - 2 x^3 y^2 + 2 x y^4 - y^5 = y(x^4 - 2 x^3 y + 2 x y^3 - y^4) = \\ & y((x^4 - y^4) - (2 x^3 y - 2 x y^3)) = y((x^2 - y^2)(x^2 + y^2) - 2 x y(x^2 - y^2)) = \\ & y(x^2 - y^2)((x^2 + y^2) - 2 x y) = y(x+y)(x-y)(x-y)^2 = y(x+y)(x-y)^3 \end{aligned}$$

$$\begin{aligned} 23) \quad & a^8 - 256 = (a^4 - 16)(a^4 + 16) = (a^2 - 4)(a^2 + 4)(a^4 + 16) = \\ & (a-2)(a+2)(a^2 + 4)(a^4 + 16) \end{aligned}$$

$$\begin{aligned} 24) \quad & a^8 - 2 a^4 + 1 = (a^4 - 1)^2 = ((a^2 - 1)(a^2 + 1))^2 = ((a-1)(a+1)(a^2 + 1))^2 = \\ & (a-1)^2(a+1)^2(a^2 + 1)^2 \end{aligned}$$

$$\begin{aligned} 25) \quad & x^6 - x^4 - 16 x^2 + 16 = x^4(x^2 - 1) - 16(x^2 - 1) = (x^2 - 1)(x^4 - 16) = \\ & (x-1)(x+1)(x^2 - 4)(x^2 + 4) = (x-1)(x+1)(x-2)(x+2)(x^2 + 4) \end{aligned}$$

$$\begin{aligned} 26) \quad & x^7 - x^4 - x^3 + 1 = x^4(x^3 - 1) - (x^3 - 1) = (x^3 - 1)(x^4 - 1) = \\ & (x-1)(x^2 + x + 1)(x^2 - 1)(x^2 + 1) = \\ & (x-1)(x^2 + x + 1)(x-1)(x+1)(x^2 + 1) = (x-1)^2(x^2 + x + 1)(x+1)(x^2 + 1) \end{aligned}$$

$$\begin{aligned} 27) \quad & (3x^2 - 7)^2 - (x^2 - 2x + 5)^2 = \\ & ((3x^2 - 7) - (x^2 - 2x + 5))((3x^2 - 7) + (x^2 - 2x + 5)) = \\ & (2x^2 + 2x - 12)(4x^2 - 2x - 2) = 2(x^2 + x - 6)2(2x^2 - x - 1) = \\ & 4(x-2)(x+3)(2x^2 - x - 1) \end{aligned}$$

Réolvons l'équation $2x^2 - x - 1 = 0$.

$$\Delta = (-1)^2 - 4 \cdot 2 \cdot (-1) = 9 = 3^2 > 0$$

$$x_1 = \frac{-(-1)-3}{2 \cdot 2} = -\frac{1}{2} \quad x_2 = \frac{-(-1)+3}{2 \cdot 2} = 1$$

$$2x^2 - x - 1 = 2\left(x + \frac{1}{2}\right)(x - 1) = (2x + 1)(x - 1)$$

On obtient donc :

$$(3x^2 - 7)^2 - (x^2 - 2x + 5)^2 = 4(x - 2)(x + 3)(2x^2 - x - 1) =$$

$$4(x - 2)(x + 3)(2x + 1)(x - 1)$$

$$28) (3x^2 + 4x - 5)^2 - (5x^2 - 4x + 3)^2 =$$

$$\left((3x^2 + 4x - 5) - (5x^2 - 4x + 3)\right) \left((3x^2 + 4x - 5) + (5x^2 - 4x + 3)\right) =$$

$$(-2x^2 + 8x - 8)(8x^2 - 2) = -2(x^2 - 4x + 4)2(4x^2 - 1) =$$

$$-4(x - 2)^2(2x - 1)(2x + 1)$$