

3.1

$$\begin{array}{r|l}
 1) & \begin{array}{r} x^4 - 3x^3 + x - 5 \\ -x^4 + 3x^2 \\ \hline -3x^3 + 3x^2 + x - 5 \\ +3x^3 - 9x \\ \hline 3x^2 - 8x - 5 \\ -3x^2 + 9 \\ \hline -8x + 4 \end{array} \\
 & \begin{array}{r} x^2 - 3 \\ \hline x^2 - 3x + 3 \end{array}
 \end{array}$$

$$x^4 - 3x^3 + x - 5 = (x^2 - 3)(x^2 - 3x + 3) + (-8x + 4)$$

$$\begin{array}{r|l}
 2) & \begin{array}{r} 35x^3 + 47x^2 + 3x + 1 \\ -35x^3 - 7x^2 \\ \hline 40x^2 + 3x + 1 \\ -40x^2 - 8x \\ \hline -5x + 1 \\ +5x + 1 \\ \hline 2 \end{array} \\
 & \begin{array}{r} 5x + 1 \\ \hline 7x^2 + 8x - 1 \end{array}
 \end{array}$$

$$35x^3 + 47x^2 + 3x + 1 = (5x + 1)(7x^2 + 8x - 1) + 2$$

$$\begin{array}{r|l}
 3) & \begin{array}{r} x^7 - 4x^6 + 2x^5 + x^4 - 3x^2 + 2x - 6 \\ -x^7 + 3x^2 \\ \hline -4x^6 + 2x^5 + x^4 + 2x - 6 \\ +4x^6 - 12x \\ \hline 2x^5 + x^4 - 10x - 6 \\ -2x^5 + 6 \\ \hline x^4 - 10x \end{array} \\
 & \begin{array}{r} x^5 - 3 \\ \hline x^2 - 4x + 2 \end{array}
 \end{array}$$

$$x^7 - 4x^6 + 2x^5 + x^4 - 3x^2 + 2x - 6 = (x^5 - 3)(x^2 - 4x + 2) + (x^4 - 10x)$$

$$\begin{array}{r|l}
 4) & \begin{array}{r} x^8 + x^4 + 1 \\ -x^8 + x^7 - x^6 \\ \hline x^7 - x^6 + x^4 + 1 \\ -x^7 + x^6 - x^5 \\ \hline -x^5 + x^4 + 1 \\ +x^5 - x^4 + x^3 \\ \hline x^3 + 1 \\ -x^3 + x^2 - x \\ \hline x^2 - x + 1 \\ -x^2 + x - 1 \\ \hline 0 \end{array} \\
 & \begin{array}{r} x^2 - x + 1 \\ \hline x^6 + x^5 - x^3 + x + 1 \end{array}
 \end{array}$$

$$x^8 + x^4 + 1 = (x^2 - x + 1)(x^6 + x^5 - x^3 + x + 1)$$

$$\begin{array}{r|l}
5) & \begin{array}{r} x^5 \qquad \qquad - 3x^3 + 2x^2 + 5x \\ - x^5 - 2x^4 \\ \hline \qquad - 2x^4 - 3x^3 + 2x^2 + 5x \\ \qquad + 2x^4 + 4x^3 \\ \hline \qquad \qquad \qquad x^3 + 2x^2 + 5x \\ \qquad \qquad - x^3 - 2x^2 \\ \hline \qquad \qquad \qquad \qquad \qquad 5x \\ \qquad \qquad \qquad - 5x - 10 \\ \hline \qquad \qquad \qquad \qquad \qquad - 10 \end{array} & \begin{array}{l} x+2 \\ \hline x^4 - 2x^3 + x^2 + 5 \end{array}
\end{array}$$

$$x^5 - 3x^3 + 2x^2 + 5x = (x+2)(x^4 - 2x^3 + x^2 + 5) - 10$$

$$\begin{array}{r|l}
6) & \begin{array}{r} x^3 \qquad \qquad \qquad + 3 \\ - x^3 + \frac{3}{5}x^2 - \frac{2}{5}x + \frac{1}{5} \\ \hline \qquad \frac{3}{5}x^2 - \frac{2}{5}x + \frac{16}{5} \end{array} & \begin{array}{l} 5x^3 - 3x^2 + 2x - 1 \\ \hline \frac{1}{5} \end{array}
\end{array}$$

$$x^3 + 3 = \frac{1}{5}(5x^3 - 3x^2 + 2x - 1) + \frac{1}{5}(3x^2 - 2x + 16)$$

$$\begin{array}{r|l}
7) & \begin{array}{r} 6x^4 + 4x^3 - 7x^2 \\ - 6x^4 \qquad \qquad + 9x^2 \\ \hline \qquad \qquad 4x^3 + 2x^2 \\ \qquad - 4x^3 \qquad \qquad + 6x \\ \hline \qquad \qquad \qquad 2x^2 + 6x \\ \qquad \qquad - 2x^2 \qquad \qquad + 3 \\ \hline \qquad \qquad \qquad \qquad 6x + 3 \end{array} & \begin{array}{l} 2x^2 - 3 \\ \hline 3x^2 + 2x + 1 \end{array}
\end{array}$$

$$6x^4 + 4x^3 - 7x^2 = (2x^2 - 3)(3x^2 + 2x + 1) + (6x + 3)$$

$$\begin{array}{r|l}
8) & \begin{array}{r} 7x^5 - x^4 + 6x^3 \qquad - 7x \\ - 7x^5 \qquad \qquad + x^3 \\ \hline \qquad - x^4 + 7x^3 \qquad - 7x \\ \qquad + x^4 \qquad \qquad - \frac{1}{7}x^2 \\ \hline \qquad \qquad \qquad 7x^3 - \frac{1}{7}x^2 - 7x \\ \qquad \qquad - 7x^3 \qquad \qquad + x \\ \hline \qquad \qquad \qquad \qquad - \frac{1}{7}x^2 - 6x \end{array} & \begin{array}{l} 7x^3 - x \\ \hline x^2 - \frac{1}{7}x + 1 \end{array}
\end{array}$$

$$7x^5 - x^4 + 6x^3 - 7x = (7x^3 - x)(x^2 - \frac{1}{7}x + 1) + (-\frac{1}{7}x^2 - 6x)$$

$$\begin{array}{r|l}
9) & \begin{array}{r} 2x^3 \qquad \qquad \qquad - 1 \\ - 2x^3 + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{2}{3} \\ \hline \qquad \frac{2}{3}x^2 - \frac{2}{3}x - \frac{5}{3} \end{array} & \begin{array}{l} 3x^3 - x^2 + x + 1 \\ \hline \frac{2}{3} \end{array}
\end{array}$$

$$2x^3 - 1 = \frac{2}{3}(3x^3 - x^2 + x + 1) + (\frac{2}{3}x^2 - \frac{2}{3}x - \frac{5}{3})$$

$$\begin{array}{r|l}
 10) & 3x^3 - x^2 + x + 1 \quad | \quad 2x^2 - 1 \\
 & - 3x^3 \qquad + \frac{3}{2}x \quad | \quad \frac{3}{2}x - \frac{1}{2} \\
 \hline
 & - x^2 + \frac{5}{2}x + 1 \\
 & + x^2 \qquad \qquad - \frac{1}{2} \\
 \hline
 & \frac{5}{2}x + \frac{1}{2}
 \end{array}$$

$$3x^3 - x^2 + x + 1 = (2x^2 - 1) \left(\frac{3}{2}x - \frac{1}{2} \right) + \left(\frac{5}{2}x + \frac{1}{2} \right)$$