

4 Fractions rationnelles

Une **fraction rationnelle** est une fraction dont le numérateur et le dénominateur sont des polynômes.

Simplification d'une fraction rationnelle

Exemples

$$1) \frac{6x^2 + 6x - 120}{4x + 20} = \frac{6(x^2 + x - 20)}{4(x + 5)} = \frac{6(x + 5)(x - 4)}{4(x + 5)} = \frac{3(x - 4)}{2}$$

$$2) \frac{x^3 + 3x^2y + 3xy^2 + y^3}{x^2 - y^2} = \frac{(x + y)^3}{(x - y)(x + y)} = \frac{(x + y)^2}{x - y}$$

4.1 Simplifier les fractions rationnelles suivantes :

$$1) \frac{x^2 - 8x + 15}{x^2 - 7x + 12} \quad 2) \frac{6x^2 + 2x}{27x^3 + 1} \quad 3) \frac{6x^2 + 11x + 6}{2x + 3}$$

$$4) \frac{x^2 - 25}{x^2 + 10x + 25} \quad 5) \frac{x - a}{ax - a^2} \quad 6) \frac{x^3 + x^2 + x + 1}{x^3 - x^2 + x - 1}$$

$$7) \frac{x^3 - 1}{x^2 - 1} \quad 8) \frac{x^4 - 4}{x^2 - 2} \quad 9) \frac{x^3 - 27}{3x^2 - 3x - 18}$$

4.2 Simplifier.

$$1) \frac{x^3 - 4x}{x^4 - x^3 - 2x^2} \quad 2) \frac{7x^3 - 3x^2 + 7x - 3}{7x - 3} \quad 3) \frac{x - 3}{4x^3 - 14x^2 + 18}$$

$$4) \frac{x^3 - x}{x^2 + x} \quad 5) \frac{x^3 + 6x^2 + 15x + 18}{x^2 - 2x - 15} \quad 6) \frac{x^3 - x^2 - 17x - 15}{x^2 - 1}$$

$$7) \frac{x^2 + x + 1}{x^2 - x + 1} \quad 8) \frac{x^2 - 10x + 21}{x^2 - 14x + 49} \quad 9) \frac{x^2 - 6x + 8}{x^3 - 5x^2 + 2x + 8}$$

Multiplication et division de fractions rationnelles

Exemples

$$1) \frac{4x^2 + 20x + 24}{x^2 + 4x - 5} \cdot \frac{x^2 - 1}{2x + 4} = \frac{4(x + 3)(x + 2)}{(x + 5)(x - 1)} \cdot \frac{(x - 1)(x + 1)}{2(x + 2)} = \frac{2(x + 3)(x + 1)}{x + 5}$$

$$2) \frac{x^4 + 2x^2y^2 + y^4}{x - y} : \frac{x^2 + y^2}{x^2 - y^2} = \frac{x^4 + 2x^2y^2 + y^4}{x - y} \cdot \frac{x^2 - y^2}{x^2 + y^2} = \frac{(x^2 + y^2)^2}{x - y} \cdot \frac{(x - y)(x + y)}{x^2 + y^2} = (x^2 + y^2)(x + y)$$

4.3 Simplifier et effectuer.

$$1) \frac{x^2 - 1}{x^2 + 6x + 9} \cdot \frac{x^2 - 9}{x + 1}$$

$$2) \frac{x^2 + ax}{x - a} \cdot \frac{ax - x^2}{x^3}$$

$$3) \frac{x^2 + 1}{x + 1} : \frac{x^4 - 1}{x^2 - 5x + 6}$$

$$4) \frac{(x + 4)(x - 3)}{x^2 - 3x} : \frac{x^2 - 6x + 9}{2x^2 - 8x + 6}$$

$$5) \frac{x^2 + xy}{x - y} \cdot \frac{xy - x^2}{x^3}$$

$$6) \frac{x^2 - xy}{x^2 + xy} \cdot \frac{x^2 - y^2}{3xy^2} \cdot \frac{x^2 y^2}{x^2 - 2xy + y^2}$$

4.4 Simplifier et effectuer.

$$1) \frac{x + 4}{x^2 + x - 12} \cdot \frac{x^2 - x - 6}{2x + 4}$$

$$2) \frac{2x^2 + 3x - 2}{x^2 - 2x + 1} \cdot \frac{x - 1}{4x^2 - 16}$$

$$3) \frac{x^3 - 3x^2 + 4x - 12}{x - 2} : \frac{x - 3}{x^2 - 4}$$

$$4) (1 + x) : \frac{x^2}{x^2 - 6x - 7}$$

$$5) \frac{x^2 - xy}{xy + y^2} : \frac{xy - y^2}{x^2 + xy}$$

$$6) \frac{xy - x + y - 1}{3x^2 + 3x} : \frac{xy - x + 2y - 2}{4x^2 + 8x}$$

Addition et soustraction de fractions rationnelles

Exemples

$$\begin{aligned} 1) \quad & \frac{2 - x}{x^2 - x - 6} + \frac{x + 1}{x^2 + 6x + 8} = \frac{2 - x}{(x - 3)(x + 2)} + \frac{x + 1}{(x + 2)(x + 4)} = \\ & \frac{(2 - x)(x + 4) + (x + 1)(x - 3)}{(x - 3)(x + 2)(x + 4)} = \frac{(2x + 8 - x^2 - 4x) + (x^2 - 3x + x - 3)}{(x - 3)(x + 2)(x + 4)} = \\ & \frac{-4x + 5}{(x - 3)(x + 2)(x + 4)} \end{aligned}$$

$$\begin{aligned} 2) \quad & \frac{x}{x^2 - y^2} - \frac{x}{x^2 + 2xy + y^2} + \frac{1}{2y - 2x} = \\ & \frac{x}{(x - y)(x + y)} - \frac{x}{(x + y)^2} - \frac{1}{2(x - y)} = \frac{2x(x + y) - 2x(x - y) - (x + y)^2}{2(x - y)(x + y)^2} = \\ & \frac{(2x^2 + 2xy) - (2x^2 - 2xy) - (x^2 + 2xy + y^2)}{2(x - y)(x + y)^2} = \frac{-x^2 + 2xy - y^2}{2(x - y)(x + y)^2} = \\ & \frac{-(x - y)^2}{2(x - y)(x + y)^2} = \frac{-(x - y)}{2(x + y)^2} = \frac{y - x}{2(x + y)^2} \end{aligned}$$

4.5 Effectuer et simplifier.

- 1) $\frac{x}{x+y} + \frac{y}{x-y} - \frac{2xy}{x^2-y^2}$
- 2) $\frac{2x+y}{3x^2} - \frac{3x+y}{2xy} + \frac{2x+y}{6y^2} - \frac{y-8x}{6xy}$
- 3) $\frac{x+y}{x-y} + \frac{x-2y}{x+y} + \frac{x^2+3y^2}{y^2-x^2}$
- 4) $\frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{x-x^3}$
- 5) $\frac{4}{x^2-y^2} + \frac{3y}{x^2y-x^3} - \frac{x-3y}{x^3-xy^2}$
- 6) $\frac{x^3-y^3}{x^2-y^2} - \frac{x^2y+xy^2}{x^2+xy}$

4.6 Effectuer et simplifier.

- 1) $\frac{3-2x}{2x+3} - \frac{2x+3}{3-2x} + \frac{36}{4x^2-9}$
- 2) $\frac{x-3}{x+3} - \frac{4x-6y}{xy+3y+2x+6} + \frac{y+6}{y+2}$
- 3) $\frac{1}{(x-y)(x-z)} + \frac{1}{(y-x)(y-z)} + \frac{1}{(z-x)(z-y)}$
- 4) $\frac{y-z}{(x-y)(x-z)} - \frac{z-x}{(y-x)(y-z)} + \frac{x-y}{(z-x)(z-y)}$
- 5) $\frac{x-3}{x^2+6x+8} - \frac{x-2}{x^2+7x+12} + \frac{5}{x^2+5x+6}$
- 6) $\frac{2x^2-x-7}{x^4-5x^2+4} - \frac{x-3}{x^3-2x^2-x+2}$

4.7 Effectuer et simplifier.

- 1) $\left(\frac{x+2}{x} - \frac{2}{x^2+x}\right) \left(\frac{1}{x} + 1\right)$
- 2) $\left[\left(x + \frac{2x}{x-2}\right) \left(\frac{2x}{x-2} - 2\right)\right] : \frac{4x^2}{x^2-4}$
- 3) $\left(\frac{1}{x} - \frac{1}{x^2}\right) : \left[\left(\frac{y}{x} + 1\right) \left(\frac{y}{x} - 1\right) \frac{x}{y^2-x^2} - \frac{1}{x^2}\right]$
- 4) $\left(\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}\right) : \left(\frac{1}{x^2} - 1\right)$
- 5) $\frac{2x-1}{x+y} \cdot \frac{x^2-y^2}{4x^2-1} \cdot \left(1 - \frac{2x}{2x-1} + \frac{1}{2x+1}\right)$
- 6) $\left(\frac{x}{x+y} + \frac{y}{x-y} - 1\right) \left(\frac{y}{x} - \frac{x}{y}\right) - \left(1 - \frac{x}{y}\right) \left(2 - \frac{2y}{x}\right)$

Réponses

- 4.1**
- | | | |
|--------------------------|---------------------------|------------------------------|
| 1) $\frac{x-5}{x-4}$ | 2) $\frac{2x}{9x^2-3x+1}$ | 3) $\frac{6x^2+11x+6}{2x+3}$ |
| 4) $\frac{x-5}{x+5}$ | 5) $\frac{1}{a}$ | 6) $\frac{x+1}{x-1}$ |
| 7) $\frac{x^2+x+1}{x+1}$ | 8) x^2+2 | 9) $\frac{x^2+3x+9}{3(x+2)}$ |
- 4.2**
- | | | |
|------------------------------|---------------------------|-----------------------------|
| 1) $\frac{x+2}{x(x+1)}$ | 2) x^2+1 | 3) $\frac{1}{2(2x^2-x-3)}$ |
| 4) $x-1$ | 5) $\frac{x^2+3x+6}{x-5}$ | 6) $\frac{(x-5)(x+3)}{x-1}$ |
| 7) $\frac{x^2+x+1}{x^2-x+1}$ | 8) $\frac{x-3}{x-7}$ | 9) $\frac{1}{x+1}$ |
- 4.3**
- | | | |
|---------------------------------|---------------------|--------------------------------------|
| 1) $\frac{(x-1)(x-3)}{x+3}$ | 2) $-\frac{x+a}{x}$ | 3) $\frac{(x-2)(x-3)}{(x-1)(x+1)^2}$ |
| 4) $\frac{2(x+4)(x-1)}{x(x-3)}$ | 5) $-\frac{x+y}{x}$ | 6) $\frac{x}{3}$ |
- 4.4**
- | | | |
|-------------------------------|-------------------------------|-------------------|
| 1) $\frac{1}{2}$ | 2) $\frac{2x-1}{4(x-1)(x-2)}$ | 3) $(x^2+4)(x+2)$ |
| 4) $\frac{(x+1)^2(x-7)}{x^2}$ | 5) $\frac{x^2}{y^2}$ | 6) $\frac{4}{3}$ |
- 4.5**
- | | | |
|---------------------------|------------------------------|----------------------|
| 1) $\frac{x-y}{x+y}$ | 2) $\frac{x^3+y^3}{3x^2y^2}$ | 3) $\frac{x}{x+y}$ |
| 4) $\frac{2}{(x-1)(x+1)}$ | 5) $\frac{3}{x^2}$ | 6) $\frac{x^2}{x+y}$ |
- 4.6**
- | | | |
|----------------------|---------------------------|---------------------------|
| 1) $\frac{12}{2x-3}$ | 2) 2 | 3) 0 |
| 4) $\frac{2}{z-x}$ | 5) $\frac{5}{(x+2)(x+4)}$ | 6) $\frac{1}{(x-2)(x+2)}$ |
- 4.7**
- | | | |
|-------------------------|------------------------------------|------------------------|
| 1) $\frac{x+3}{x}$ | 2) $\frac{x+2}{x-2}$ | 3) 1 |
| 4) $\frac{x-3}{x(1-x)}$ | 5) $\frac{2(y-x)}{(2x-1)(2x+1)^2}$ | 6) $\frac{2(x-2y)}{y}$ |

4.8

Calculer :

$$1) \frac{1 - \frac{1}{1-x}}{1 - \frac{1}{1+x}}$$

$$2) \frac{\frac{x}{x+y} + \frac{y}{x-y}}{\frac{x}{x-y} - \frac{y}{x+y}}$$

$$3) \frac{\frac{x^2}{y} + \frac{y^2}{x}}{\frac{x}{y} + \frac{y}{x} - 1}$$

$$4) \frac{\frac{1}{x} + 1}{\frac{1}{x}} \cdot \frac{\frac{1}{x} + x - 1}{x^2 - x + 1}$$

$$5) \frac{\frac{x}{x+1} - \frac{x}{1-x}}{1 + \frac{1}{x^2 - 1}}$$

$$6) \frac{1 - \frac{x-y}{x+y}}{\frac{x-y}{x+y} - \frac{x+y}{x-y}}$$

$$7) \frac{\frac{1}{x+1} - \frac{1}{x-1}}{\frac{1}{x-1} + \frac{1}{x+1}}$$

$$8) \frac{\frac{y^2 - x^2}{x y^2 - x^2 y}}{\frac{x+y}{x y}}$$

4.9

Vérifier les égalités :

$$1) \frac{\frac{x+1}{x} \cdot \left(\frac{x^2}{x+2} - x + 2 \right)}{\frac{2x}{x^2 + 3x + 2}} = \frac{2(x+1)^2}{x^2}$$

$$2) \frac{\frac{1}{y} + \frac{y}{x}}{\frac{x^2 - y^4}{2x^2 y^2}} - \frac{1 + \frac{y^2}{x^2}}{\frac{1}{x} - \frac{y^2}{x^2}} = -\frac{(x-y)^2}{x-y^2}$$

$$3) \left(\frac{1}{y} - \frac{1}{x} \right)^2 + \frac{\left(\frac{1}{y^2} - \frac{1}{x^2} \right) \frac{4y}{x^2 - y^2}}{1 - \frac{x-1}{x}} = \left(\frac{x+y}{xy} \right)^2$$

$$4) \frac{\frac{2x+1}{x^2 - 3x + 2} - \frac{x+3}{x-2} + \frac{x+1}{x-1}}{x - \frac{x^2 - x - 1}{x-1}} = -1$$

$$5) \frac{\frac{2}{x} - \frac{3}{y}}{\frac{2}{x} + \frac{3}{y}} = \frac{\frac{4}{x^2} - \frac{9}{y^2}}{\frac{4}{x^2} + \frac{9}{y^2} + \frac{12}{xy}}$$

$$6) \quad \frac{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}{\frac{1}{xy} + \frac{1}{xz} + \frac{1}{yz}} \cdot \frac{x+y+z}{xyz+xz} = \frac{x+3y+5z}{\frac{5}{xy} + \frac{3}{xz} + \frac{1}{yz}} \cdot \frac{1}{xyz}$$

$$7) \quad \frac{\frac{2y+3}{y+1} - \frac{1}{y+2}}{y+2} = \left(\frac{1}{y^2+2y+1} + \frac{1}{y^2+4y+4} \right) (y+1)$$

$$8) \quad (x+y+z) \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = \frac{(x+y)(y+z)(x+z)}{xyz} + 1$$

$$9) \quad \frac{1}{x-1} + \frac{1}{x^2-1} \left(\frac{1+x}{1-x} + \frac{x \left(1 + \frac{1}{x} \right)}{1 - \frac{1}{x}} \right) = \frac{\frac{2}{x^2-1}}{\frac{1}{x+1}}$$

Réponses

4.8	1) $\frac{x+1}{x-1}$	2) 1	3) $x+y$
	4) $\frac{1+x}{x}$	5) 2	6) $\frac{y-x}{2x}$
	7) $-\frac{1}{x}$	8) 1	