

4.6

$$\begin{aligned}
1) \quad & \frac{3-2x}{2x+3} - \frac{2x+3}{3-2x} + \frac{36}{4x^2-9} = \frac{3-2x}{2x+3} + \frac{2x+3}{2x-3} + \frac{36}{(2x-3)(2x+3)} \\
&= \frac{(3-2x)(2x-3) + (2x+3)^2 + 36}{(2x-3)(2x+3)} \\
&= \frac{6x-9-4x^2+6x+4x^2+12x+9+36}{(2x-3)(2x+3)} \\
&= \frac{24x+36}{(2x-3)(2x+3)} = \frac{12(2x+3)}{(2x-3)(2x+3)} = \frac{12}{2x-3}
\end{aligned}$$

$$\begin{aligned}
2) \quad & \frac{x-3}{x+3} - \frac{4x-6y}{xy+3y+2x+6} + \frac{y+6}{y+2} = \frac{x-3}{x+3} - \frac{4x-6y}{y(x+3)+2(x+3)} + \frac{y+6}{y+2} \\
&= \frac{x-3}{x+3} - \frac{4x-6y}{(x+3)(y+2)} + \frac{y+6}{y+2} \\
&= \frac{(x-3)(y+2) - (4x-6y) + (y+6)(x+3)}{(x+3)(y+2)} \\
&= \frac{xy+2x-3y-6-4x+6y+xy+3y+6x+18}{(x+3)(y+2)} \\
&= \frac{2xy+4x+6y+12}{(x+3)(y+2)} = \frac{2(xy+2x+3y+6)}{(x+3)(y+2)} \\
&= \frac{2(x(y+2)+3(y+2))}{(x+3)(y+2)} = \frac{2(y+2)(x+3)}{(x+3)(y+2)} = 2
\end{aligned}$$

$$\begin{aligned}
3) \quad & \frac{1}{(x-y)(x-z)} + \frac{1}{(y-x)(y-z)} + \frac{1}{(z-x)(z-y)} \\
&= \frac{1}{(x-y)(x-z)} - \frac{1}{(x-y)(y-z)} + \frac{1}{(x-z)(y-z)} \\
&= \frac{(y-z) - (x-z) + (x-y)}{(x-y)(x-z)(y-z)} = \frac{y-z-x+z+x-y}{(x-y)(x-z)(y-z)} = 0
\end{aligned}$$

$$\begin{aligned}
4) \quad & \frac{y-z}{(x-y)(x-z)} - \frac{z-x}{(y-x)(y-z)} + \frac{x-y}{(z-x)(z-y)} \\
&= \frac{y-z}{(x-y)(x-z)} + \frac{z-x}{(x-y)(y-z)} + \frac{x-y}{(x-z)(y-z)} \\
&= \frac{(y-z)^2 + (z-x)(x-z) + (x-y)^2}{(x-y)(x-z)(y-z)} \\
&= \frac{y^2 - 2yz + z^2 + xz - z^2 - x^2 + xz + x^2 - 2xy + y^2}{(x-y)(x-z)(y-z)} \\
&= \frac{2y^2 - 2xy + 2xz - 2yz}{(x-y)(x-z)(y-z)} = \frac{2(y^2 - xy + xz - yz)}{(x-y)(x-z)(y-z)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(y(y-x) + z(x-y))}{(x-y)(x-z)(y-z)} = \frac{2(-y(x-y) + z(x-y))}{(x-y)(x-z)(y-z)} \\
&= \frac{2(x-y)(-y+z)}{(x-y)(x-z)(y-z)} = \frac{-2(x-y)(y-z)}{(x-y)(x-z)(y-z)} \\
&= \frac{-2}{x-z} = \frac{2}{z-x}
\end{aligned}$$

$$\begin{aligned}
5) \quad & \frac{x-3}{x^2+6x+8} - \frac{x-2}{x^2+7x+12} + \frac{5}{x^2+5x+6} \\
&= \frac{x-3}{(x+2)(x+4)} - \frac{x-2}{(x+3)(x+4)} + \frac{5}{(x+2)(x+3)} \\
&= \frac{(x-3)(x+3) - (x-2)(x+2) + 5(x+4)}{(x+2)(x+3)(x+4)} \\
&= \frac{x^2-9-x^2+4+5x+20}{(x+2)(x+3)(x+4)} = \frac{5x+15}{(x+2)(x+3)(x+4)} \\
&= \frac{5(x+3)}{(x+2)(x+3)(x+4)} = \frac{5}{(x+2)(x+4)}
\end{aligned}$$

$$\begin{aligned}
6) \quad & \frac{2x^2-x-7}{x^4-5x^2+4} - \frac{x-3}{x^3-2x^2-x+2} = \frac{2x^2-x-7}{(x^2-1)(x^2-4)} - \frac{x-3}{x^2(x-2)-(x-2)} \\
&= \frac{2x^2-x-7}{(x-1)(x+1)(x-2)(x+2)} - \frac{x-3}{(x-2)\underbrace{(x^2-1)}_{(x-1)(x+1)}} \\
&= \frac{(2x^2-x-7)-(x-3)(x+2)}{(x-1)(x+1)(x-2)(x+2)} \\
&= \frac{2x^2-x-7-x^2-2x+3x+6}{(x-1)(x+1)(x-2)(x+2)} \\
&= \frac{x^2-1}{(x-1)(x+1)(x-2)(x+2)} = \frac{(x-1)(x+1)}{(x-1)(x+1)(x-2)(x+2)} \\
&= \frac{1}{(x-2)(x+2)}
\end{aligned}$$