

5.10

$$1) \begin{cases} 4x - 4 \leq 0 \\ 3x + 2 > 0 \end{cases}$$

$$\begin{cases} 4x \leq 4 \\ 3x > -2 \end{cases}$$

$$\begin{cases} x \leq 1 \\ x > -\frac{2}{3} \end{cases}$$

$$S = (]-\infty; 1]) \cap (]-\frac{2}{3}; +\infty[) =]-\frac{2}{3}; 1]$$

$$2) \begin{cases} x^2 - 10x + 21 \leq 0 \\ -x^2 - 6x + 16 > 0 \end{cases}$$

$$\begin{cases} x^2 - 10x + 21 \leq 0 \\ x^2 + 6x - 16 < 0 \end{cases}$$

$$\begin{cases} (x - 3)(x - 7) \leq 0 \\ (x - 2)(x + 8) < 0 \end{cases}$$

$x - 3$	$\overset{3}{-}$	$\overset{7}{0}$	$+$	$+$
$x - 7$	$-$	0	$-$	$+$
$P(x)$	$+$	0	$-$	0

$x - 2$	$\overset{-8}{-}$	$\overset{2}{0}$	$-$	$+$
$x + 8$	$-$	0	$-$	$+$
$P(x)$	$+$	0	$-$	0

$$S = ([3; 7]) \cap (]-8; 2]) = \emptyset$$

$$3) \begin{cases} \frac{3x - 8}{5 - 6x} \geq 0 \\ \frac{x^2 + 1}{x + 1} > 0 \end{cases}$$

$3x - 8$	$-$	$\parallel$	$\overset{\frac{5}{6}}{-}$	$\overset{\frac{8}{3}}{0}$	$+$
$5 - 6x$	$+$	$\parallel$	$-$	0	$-$
$F(x)$	$-$	$\parallel$	$+$	0	$-$

$x^2 + 1$	$+$	$\parallel$	$\overset{-1}{+}$
$x + 1$	$-$	$\parallel$	$+$
$F(x)$	$-$	$\parallel$	$+$

$$S = (]\frac{5}{6}; \frac{8}{3}]) \cap (]-1; +\infty[) =]\frac{5}{6}; \frac{8}{3}]$$

$$4) \begin{cases} 4x - 4 \leq 0 \\ 3x + 12 < 0 \\ x + 8 > 3x + 2 \end{cases}$$

$$\begin{cases} 4x \leq 4 \\ 3x < -12 \\ -2x > -6 \end{cases}$$

$$\begin{cases} x \leq 1 \\ x < -4 \\ x < 3 \end{cases}$$

$$S = (]-\infty; 1]) \cap (]-\infty; -4[) \cap (]-\infty; -3[) =]-\infty; -4[$$

$$5) -3 < \frac{7-x}{x+3} \leq 3$$

$$\begin{cases} -3 < \frac{7-x}{x+3} \\ \frac{7-x}{x+3} \leq 3 \end{cases}$$

$$\begin{cases} 0 < \frac{7-x}{x+3} + 3 \\ \frac{7-x}{x+3} - 3 \leq 0 \end{cases}$$

$$\begin{cases} 0 < \frac{(7-x) + 3(x+3)}{x+3} \\ \frac{(7-x) - 3(x+3)}{x+3} \leq 0 \end{cases}$$

$$\begin{cases} 0 < \frac{2x+16}{x+3} \\ \frac{-4x-2}{x+3} \leq 0 \end{cases}$$

$$\begin{array}{c|ccc|ccc} & & -8 & & -3 & & & \\ \hline \frac{2x+16}{x+3} & - & 0 & + & || & + & & \\ \hline F(x) & + & 0 & - & || & + & & \end{array}$$

$$\begin{array}{c|ccc|ccc} & & -3 & & -\frac{1}{2} & & & \\ \hline \frac{-4x-2}{x+3} & + & || & + & 0 & - & & \\ \hline F(x) & - & || & + & 0 & - & & \end{array}$$

$$S = (]-\infty; -8[\cup]-3; +\infty[) \cap (]-\infty; -3[\cup [-\frac{1}{2}; +\infty[) \\ =]-\infty; -8[\cup [-\frac{1}{2}; +\infty[$$

$$6) (x-2) \left(\frac{2x+3}{2-x} - 4x \right) \leq 1$$

$$(x-2) \cdot \frac{(2x+3) - 4x(2-x)}{2-x} \leq 1$$

$$(x-2) \cdot \frac{4x^2 - 6x + 3}{2-x} \leq 1$$

$$\frac{(x-2)(4x^2 - 6x + 3)}{2-x} - 1 \leq 0$$

$$\frac{(x-2)(4x^2-6x+3)-1(2-x)}{2-x} \leq 0$$

$$\frac{(x-2)(4x^2-6x+3)+1(x-2)}{2-x} \leq 0$$

$$\frac{(x-2)(4x^2-6x+4)}{2-x} \leq 0$$

$x-2$	$-$	$+$
$4x^2-6x+4$	$+$	$+$
$2-x$	$+$	$-$
$F(x)$	$-$	$-$

$$\Delta = (-6)^2 - 4 \cdot 4 \cdot 4 = -28 < 0$$

$$S = \mathbb{R} - \{2\} =]-\infty; 2[\cup]2; +\infty[$$

$$7) \begin{cases} x^2 - 4 \leq 0 \\ 3x + 2 > 0 \end{cases}$$

$$\begin{cases} (x-2)(x+2) \leq 0 \\ x > -\frac{2}{3} \end{cases}$$

$x-2$	$-$	$-$	0	$+$
$x+2$	$-$	0	$+$	$+$
$P(x)$	$+$	0	$-$	0

$$S = [-2; 2] \cap]-\frac{2}{3}; +\infty[=]-\frac{2}{3}; 2]$$

$$8) \begin{cases} 4x^2 - 4x \leq -1 \\ \frac{x+7}{2} > x-3 \end{cases}$$

$$\begin{cases} 4x^2 - 4x + 1 \leq 0 \\ x+7 > 2x-6 \end{cases}$$

$$\begin{cases} (2x-1)^2 \leq 0 \\ 13 > x \end{cases}$$

$$S = \{\frac{1}{2}\} \cap]-\infty; 13[= \{\frac{1}{2}\}$$

$$9) \begin{cases} \frac{3x-5}{2-6x} \geq 0 \\ \frac{x^2+1}{(x+1)^2} > 1 \end{cases}$$

$$\begin{cases} \frac{3x-5}{2-6x} \geq 0 \\ \frac{x^2+1}{(x+1)^2} - 1 > 0 \end{cases}$$

$$\begin{cases} \frac{3x-5}{2-6x} \geq 0 \\ \frac{(x^2+1) - 1(x+1)^2}{(x+1)^2} > 0 \end{cases}$$

$$\begin{cases} \frac{3x-5}{2-6x} \geq 0 \\ \frac{-2x}{(x+1)^2} > 0 \end{cases}$$

$3x-5$	-	$\frac{1}{3}$	-	$\frac{5}{3}$	+
$2-6x$	+		-		-
$F(x)$	-		+		-

$-2x$	+	-1	+	0	-
$(x+1)^2$	+		+		+
$F(x)$	+		+		-

$$S =]\frac{1}{3}; \frac{5}{3}] \cap (]-\infty; -1[\cup]-1; 0[) = \emptyset$$

10) $-3 < 1 - x^2 \leq 3$

$$\begin{cases} -3 < 1 - x^2 \\ 1 - x^2 \leq 3 \end{cases}$$

$$\begin{cases} 0 < 4 - x^2 \\ 0 \leq x^2 + 2 \end{cases}$$

$$\begin{cases} 0 < (2-x)(2+x) \\ 0 \leq x^2 + 2 \end{cases}$$

$2-x$	+	-2	+	2	-
$2+x$	-	0	+	0	+
$P(x)$	-	0	+	0	-

$$x^2 + 2 \mid + \quad \Delta = 0^2 - 4 \cdot 1 \cdot 2 = -8 < 0$$

$$S =]-2; 2[\cap \mathbb{R} =]-2; 2[$$

11)
$$\begin{cases} 9x - 2 < 3x - 14 \\ 5x + 3 < 15 - 7x \\ 11x - 4 > 10x - 9 \end{cases}$$

$$\begin{cases} 6x < -12 \\ 12x < 12 \\ x > -5 \end{cases}$$

$$\begin{cases} x < -2 \\ x < 1 \\ x > -5 \end{cases}$$

$$S = (]-\infty; -2[) \cap (]-\infty; 1[) \cap (]-5; +\infty[) =]-5; -2[$$

$$12) \begin{cases} \frac{4x-5}{2-3x} \geq 0 \\ \frac{x^2+4}{x+2} > 0 \end{cases}$$

$$\begin{array}{c|c|c|c|c} \frac{4x-5}{2-3x} & - & \frac{\frac{2}{3}}{\parallel} & - & \frac{\frac{5}{4}}{0} & + \\ \hline & + & \parallel & - & & - \\ \hline F(x) & - & \parallel & + & 0 & - \end{array}$$

$$\begin{array}{c|c|c|c} \frac{x^2+4}{x+2} & + & \frac{-2}{\parallel} & + \\ \hline & - & \parallel & + \\ \hline F(x) & - & \parallel & + \end{array}$$

$$S = (]\frac{2}{3}; \frac{5}{4}]) \cap (]-2; +\infty[) =]\frac{2}{3}; \frac{5}{4}]$$

$$13) \begin{cases} (x+1)^3 - 3(x-1)^2 > x^3 - 11 \\ \frac{3x-1}{2} < x + \frac{x-2}{5} \\ 1 + \frac{2}{x} \leq \frac{3}{x^2} \end{cases}$$

$$\begin{cases} x^3 + 3x^2 + 3x + 1 - 3x^2 + 6x - 3 > x^3 - 11 \\ 5(3x-1) < 10x + 2(x-2) \\ 0 \leq 1 + \frac{2}{x} + \frac{3}{x^2} \end{cases}$$

$$\begin{cases} x^3 + 9x - 2 > x^3 - 11 \\ 15x - 5 < 10x + 2x - 4 \\ 0 \leq \frac{x^2 + 2x + 3}{x^2} \end{cases}$$

$$\begin{cases} 9x > -9 \\ 3x < 1 \\ 0 \leq \frac{x^2 + 2x + 3}{x^2} \end{cases}$$

$$\begin{cases} x > -1 \\ x < \frac{1}{3} \\ 0 \leq \frac{x^2 + 2x + 3}{x^2} \end{cases}$$

$x^2 + 2x + 3$	+	0	+
x^2	+		+
$F(x)$	+		+

 $\Delta = 2^2 - 4 \cdot 1 \cdot 3 = -8 < 0$

$$S = (]-1; +\infty[) \cap (]-\infty; \frac{1}{3}[) \cap (\mathbb{R} - \{0\}) =]-1; 0[\cup]0; \frac{1}{3}[$$

$$14) \begin{cases} (x+2)(6-x) \geq 0 \\ x^2 - x - 12 \leq 0 \\ \frac{x}{x^2 - 9} > 0 \end{cases}$$

$$\begin{cases} (x+2)(6-x) \geq 0 \\ (x-4)(x+3) \leq 0 \\ \frac{x}{(x-3)(x+3)} > 0 \end{cases}$$

		-2		6	
$x+2$	-	0	+		+
$6-x$	+		+	0	-
$P(x)$	-	0	+	0	-

		-3		4	
$x-4$	-		-	0	+
$x+3$	-	0	+		+
$P(x)$	+	0	-	0	+

		-3		0		3	
x	-		-	0	+		+
$x-3$	-		-		-		+
$x+3$	-		+		+		+
$F(x)$	-		+	0	-		+

$$S = ([-2; 6]) \cap ([-3; 4]) \cap (]-3; 0[\cup]3; +\infty[) = [-2; 0[\cup]3; 4]$$