

12.1 Soient $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ trois vecteurs du plan et $\lambda \in \mathbb{R}$.

$$1) \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 = b_1 a_1 + b_2 a_2 = \vec{b} \cdot \vec{a}$$

$$\begin{aligned} 2) \vec{a} \cdot (\vec{b} + \vec{c}) &= \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \left(\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \right) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \end{pmatrix} = \\ &= a_1 (b_1 + c_1) + a_2 (b_2 + c_2) = a_1 b_1 + a_1 c_1 + a_2 b_2 + a_2 c_2 = \\ &= (a_1 b_1 + a_2 b_2) + (a_1 c_1 + a_2 c_2) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \end{aligned}$$

$$\begin{aligned} 3) (\lambda \vec{a}) \cdot \vec{b} &= \left(\lambda \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \right) \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \lambda a_1 \\ \lambda a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \lambda a_1 b_1 + \lambda a_2 b_2 = \\ &= \lambda (a_1 b_1 + a_2 b_2) = \lambda (\vec{a} \cdot \vec{b}) \end{aligned}$$

$$4) \vec{a} \cdot \vec{a} = a_1 a_1 + a_2 a_2 = a_1^2 + a_2^2 = \left(\sqrt{a_1^2 + a_2^2} \right)^2 = \|\vec{a}\|^2$$

Soient $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ trois vecteurs de l'espace et $\lambda \in \mathbb{R}$.

$$1) \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = b_1 a_1 + b_2 a_2 + b_3 a_3 = \vec{b} \cdot \vec{a}$$

$$\begin{aligned} 2) \vec{a} \cdot (\vec{b} + \vec{c}) &= \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \left(\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \right) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \\ b_3 + c_3 \end{pmatrix} = \\ &= a_1 (b_1 + c_1) + a_2 (b_2 + c_2) + a_3 (b_3 + c_3) = a_1 b_1 + a_1 c_1 + a_2 b_2 + a_2 c_2 + a_3 b_3 + a_3 c_3 = \\ &= (a_1 b_1 + a_2 b_2 + a_3 b_3) + (a_1 c_1 + a_2 c_2 + a_3 c_3) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \end{aligned}$$

$$\begin{aligned} 3) (\lambda \vec{a}) \cdot \vec{b} &= \left(\lambda \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right) \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} \lambda a_1 \\ \lambda a_2 \\ \lambda a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \lambda a_1 b_1 + \lambda a_2 b_2 + \lambda a_3 b_3 = \\ &= \lambda (a_1 b_1 + a_2 b_2 + a_3 b_3) = \lambda (\vec{a} \cdot \vec{b}) \end{aligned}$$

$$4) \vec{a} \cdot \vec{a} = a_1 a_1 + a_2 a_2 + a_3 a_3 = a_1^2 + a_2^2 + a_3^2 = \left(\sqrt{a_1^2 + a_2^2 + a_3^2} \right)^2 = \|\vec{a}\|^2$$