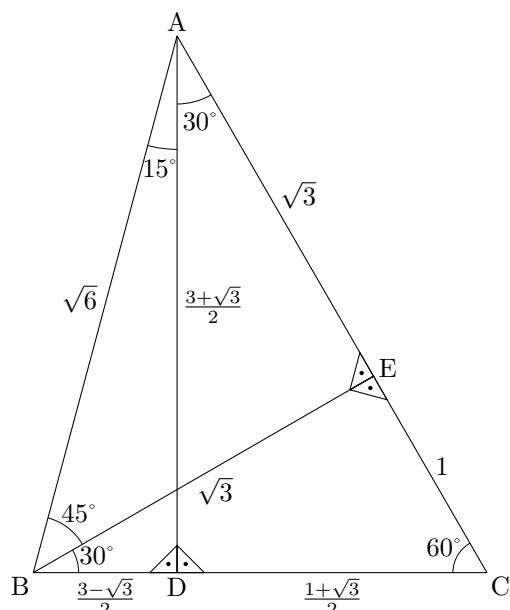


## 15.3



- 1) Considérons le triangle BCE.

Comme  $\tan(\widehat{CBE}) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ , on sait, par l'exercice 15.1, que  $\widehat{CBE} = 30^\circ$ .

Dès lors,  $\widehat{ECB} = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$ .

Examinons le triangle ABE.

Il est rectangle et isocèle en E, car  $AE = BE = \sqrt{3}$ .

C'est pourquoi  $\widehat{EBA} = \frac{1}{2} (180^\circ - 90^\circ) = 45^\circ$ .

En prenant tour à tour en compte les triangles ABD et ACD, on obtient d'une part  $\widehat{BAD} = 180^\circ - (90^\circ + 30^\circ + 45^\circ) = 15^\circ$  et d'autre part  $\widehat{DAC} = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$ .

$$2) AB = \sqrt{AE^2 + BE^2} = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{3 + 3} = \sqrt{6}$$

$$BC = \sqrt{BE^2 + CE^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$CD = AC \cdot \cos(60^\circ) = (1 + \sqrt{3}) \cdot \frac{1}{2} = \frac{1 + \sqrt{3}}{2}$$

$$AD = AC \cdot \sin(60^\circ) = (1 + \sqrt{3}) \cdot \frac{\sqrt{3}}{2} = \frac{3 + \sqrt{3}}{2}$$

$$BD = BC - CD = 2 - \frac{1 + \sqrt{3}}{2} = \frac{4 - 1 - \sqrt{3}}{2} = \frac{3 - \sqrt{3}}{2}$$

$$\begin{aligned} 3) \cos(15^\circ) &= \frac{AD}{AB} = \frac{\frac{3 + \sqrt{3}}{2}}{\sqrt{6}} = \frac{3 + \sqrt{3}}{2\sqrt{6}} = \frac{(3 + \sqrt{3}) \cdot \sqrt{6}}{2 \cdot 6} = \frac{3\sqrt{6} + \sqrt{18}}{12} = \\ &= \frac{3\sqrt{6} + 3\sqrt{2}}{12} = \frac{3(\sqrt{6} + \sqrt{2})}{12} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \sin(15^\circ) &= \frac{BD}{AB} = \frac{\frac{3 - \sqrt{3}}{2}}{\sqrt{6}} = \frac{3 - \sqrt{3}}{2\sqrt{6}} = \frac{(3 - \sqrt{3}) \cdot \sqrt{6}}{2 \cdot 6} = \frac{3\sqrt{6} - \sqrt{18}}{12} = \\ &= \frac{3\sqrt{6} - 3\sqrt{2}}{12} = \frac{3(\sqrt{6} - \sqrt{2})}{12} = \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned}\tan(15^\circ) &= \frac{BD}{AD} = \frac{\frac{3-\sqrt{3}}{2}}{\frac{3+\sqrt{3}}{2}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{(3-\sqrt{3})^2}{(3+\sqrt{3})(3-\sqrt{3})} = \frac{9-6\sqrt{3}+3}{9-3} = \\ &= \frac{12-6\sqrt{3}}{6} = \frac{6(2-\sqrt{3})}{6} = 2-\sqrt{3}\end{aligned}$$

$$\cos(75^\circ) = \frac{BD}{AB} = \sin(15^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin(75^\circ) = \frac{AD}{AB} = \cos(15^\circ) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\begin{aligned}\tan(75^\circ) &= \frac{AD}{BD} = \frac{\frac{3+\sqrt{3}}{2}}{\frac{3-\sqrt{3}}{2}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{9+6\sqrt{3}+3}{9-3} = \\ &= \frac{12+6\sqrt{3}}{6} = \frac{6(2+\sqrt{3})}{6} = 2+\sqrt{3}\end{aligned}$$