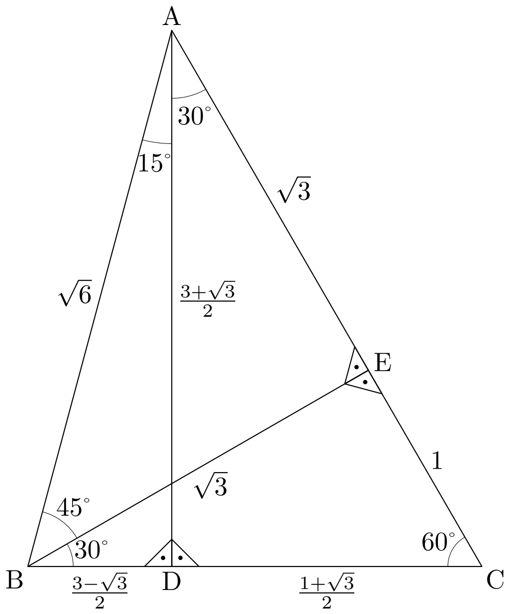
16.10



1) Considérons le triangle BCE.
Comme $\tan(\widehat{\text{CBE}}) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, on sait, par l'exercice 15.1, que $\widehat{\text{CBE}} = 30°$.
Dès lors, $\widehat{\text{ECB}} = 180° - (90° + 30°) = 60°$.

Examinons le triangle ABE.
Il est rectangle et isocèle en E, car $\text{AE} = \text{BE} = \sqrt{3}$.
C'est pourquoi $\widehat{\text{EBA}} = \frac{1}{2}\,(180° - 90°) = 45°$.

En prenant tour à tour en compte les triangles ABD et ACD, on obtient d'une part $\widehat{\text{BAD}} = 180° - (90° + 30° + 45°) = 15°$ et d'autre part $\widehat{\text{DAC}} = 180° - (90° + 60°) = 30°$.

2) $\text{AB} = \sqrt{\text{AE}^2 + \text{BE}^2} = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{3+3} = \sqrt{6}$

$\text{BC} = \sqrt{\text{BE}^2 + \text{CE}^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\text{CD} = \text{AC} \cdot \cos(60°) = (1+\sqrt{3}) \cdot \frac{1}{2} = \frac{1+\sqrt{3}}{2}$

$\text{AD} = \text{AC} \cdot \sin(60°) = (1+\sqrt{3}) \cdot \frac{\sqrt{3}}{2} = \frac{3+\sqrt{3}}{2}$

$\text{BD} = \text{BC} - \text{CD} = 2 - \frac{1+\sqrt{3}}{2} = \frac{4-1-\sqrt{3}}{2} = \frac{3-\sqrt{3}}{2}$

3) $$\cos(15°) = \frac{\text{AD}}{\text{AB}} = \frac{\frac{3+\sqrt{3}}{2}}{\sqrt{6}} = \frac{3+\sqrt{3}}{2\sqrt{6}} = \frac{(3+\sqrt{3})\cdot\sqrt{6}}{2\cdot 6} = \frac{3\sqrt{6}+\sqrt{18}}{12} =$$
$$= \frac{3\sqrt{6}+3\sqrt{2}}{12} = \frac{3\,(\sqrt{6}+\sqrt{2})}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin(15°) = \frac{\text{BD}}{\text{AB}} = \frac{\frac{3-\sqrt{3}}{2}}{\sqrt{6}} = \frac{3-\sqrt{3}}{2\sqrt{6}} = \frac{(3-\sqrt{3})\cdot\sqrt{6}}{2\cdot 6} = \frac{3\sqrt{6}-\sqrt{18}}{12} =$$
$$= \frac{3\sqrt{6}-3\sqrt{2}}{12} = \frac{3\,(\sqrt{6}-\sqrt{2})}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\tan(15^\circ) = \frac{\text{BD}}{\text{AD}} = \frac{\frac{3-\sqrt{3}}{2}}{\frac{3+\sqrt{3}}{2}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{(3-\sqrt{3})^2}{(3+\sqrt{3})\,(3-\sqrt{3})} = \frac{9-6\,\sqrt{3}+3}{9-3} =$$

$$= \frac{12-6\,\sqrt{3}}{6} = \frac{6\,(2-\sqrt{3})}{6} = 2-\sqrt{3}$$

$$\cos(75^\circ) = \frac{\text{BD}}{\text{AB}} = \sin(15^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin(75^\circ) = \frac{\text{AD}}{\text{AB}} = \cos(15^\circ) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan(75^\circ) = \frac{\text{AD}}{\text{BD}} = \frac{\frac{3+\sqrt{3}}{2}}{\frac{3-\sqrt{3}}{2}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{(3-\sqrt{3})\,(3+\sqrt{3})} = \frac{9+6\,\sqrt{3}+3}{9-3} =$$

$$= \frac{12+6\,\sqrt{3}}{6} = \frac{6\,(2+\sqrt{3})}{6} = 2+\sqrt{3}$$