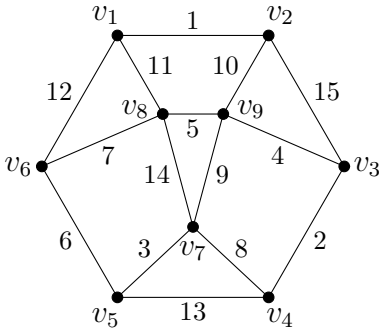
3.11 Algorithme de Kruskal

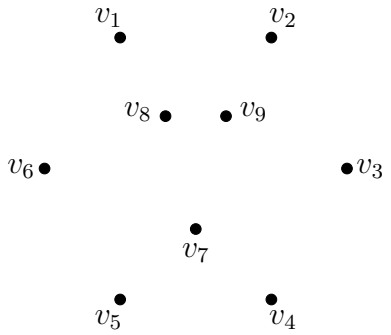


Ordonnons les arêtes du graphe par ordre croissant de poids :

$$\underbrace{v_1\,v_2}_{1},\ \underbrace{v_3\,v_4}_{2},\ \underbrace{v_5\,v_7}_{3},\ \underbrace{v_3\,v_9}_{4},\ \underbrace{v_8\,v_9}_{5},\ \underbrace{v_5\,v_6}_{6},\ \underbrace{v_6\,v_8}_{7},\ \underbrace{v_4\,v_7}_{8},\ \underbrace{v_7\,v_9}_{9},\ \underbrace{v_2\,v_9}_{10},\ \underbrace{v_1\,v_8}_{11},\ \underbrace{v_1\,v_6}_{12},\ \underbrace{v_4\,v_5}_{13},\ \underbrace{v_7\,v_8}_{14},\ \underbrace{v_2\,v_3}_{15}$$

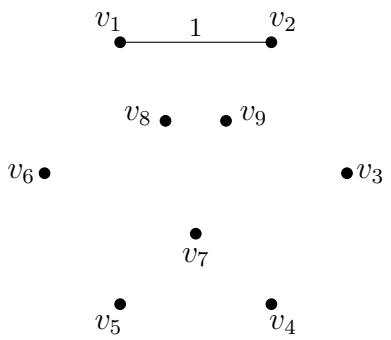
Appliquons les différentes étapes de l'algorithme de Kruskal :

1)



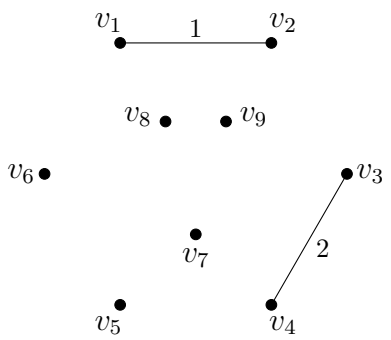
$\mathrm{F} = \varnihil$
Ajoutons l'arête $v_1\,v_2$.

2)



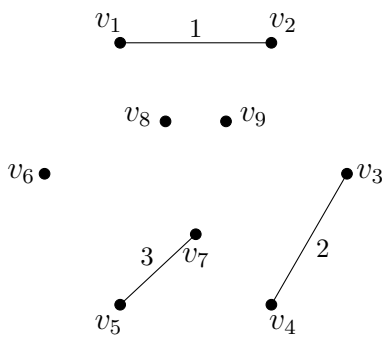
$\mathrm{F} = \{v_1\,v_2\}$
Ajoutons l'arête $v_3\,v_4$.

3)



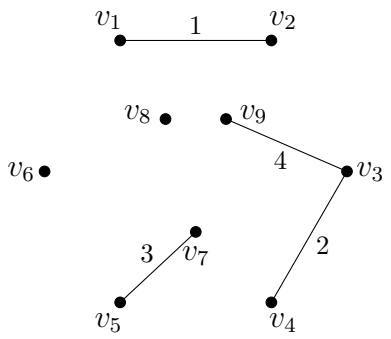
$\mathrm{F} = \{v_1\,v_2, v_3\,v_4\}$
Ajoutons l'arête $v_5\,v_7$.

4)



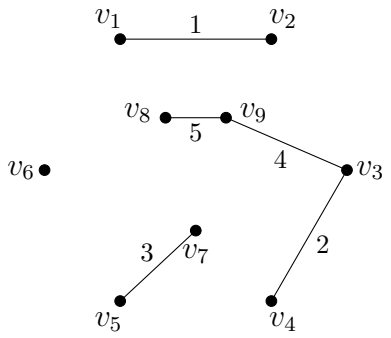
F = $\{v_1\,v_2, v_3\,v_4, v_5\,v_7\}$
Ajoutons l'arête $v_3\,v_9$.

5)



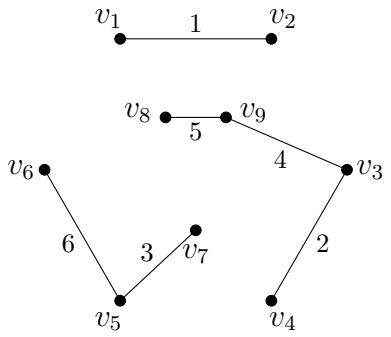
F = $\{v_1\,v_2, v_3\,v_4, v_5\,v_7, v_3\,v_9\}$
Ajoutons l'arête $v_8\,v_9$.

6)



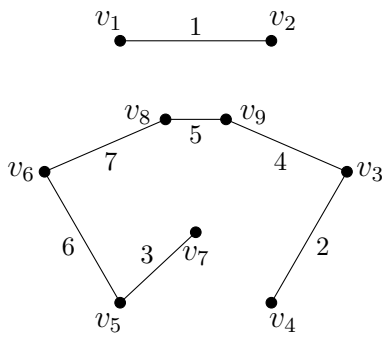
F = $\{v_1\,v_2, v_3\,v_4, v_5\,v_7, v_3\,v_9, v_8\,v_9\}$
Ajoutons l'arête $v_5\,v_6$.

7)



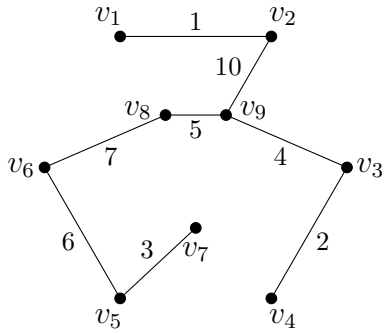
F = $\{v_1\,v_2, v_3\,v_4, v_5\,v_7, v_3\,v_9, v_8\,v_9,$
$v_5\,v_6\}$
Ajoutons l'arête $v_6\,v_8$.

8)



F = $\{v_1\,v_2, v_3\,v_4, v_5\,v_7, v_3\,v_9, v_8\,v_9, v_5\,v_6, v_6\,v_8\}$
L'ajout de l'arête $v_4\,v_7$ créerait le cycle $v_3\,v_4\,v_7\,v_5\,v_6\,v_8\,v_9\,v_3$.
L'ajout de l'arête $v_7\,v_9$ produirait le cycle $v_5\,v_6\,v_8\,v_9\,v_7\,v_5$.
Ajoutons l'arête $v_2\,v_9$.

9)

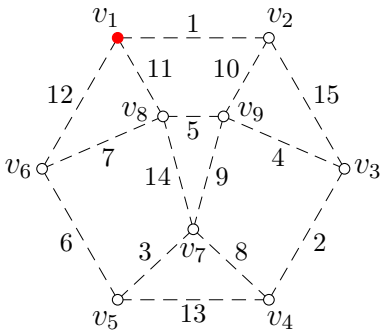


F = $\{v_1\,v_2, v_3\,v_4, v_5\,v_7, v_3\,v_9, v_8\,v_9, v_5\,v_6, v_6\,v_8, v_2\,v_9\}$
À présent que tous les sommets sont connectés, nous avons obtenu l'arbre de recouvrement de poids minimum.

Algorithme de Prim

Voici les différentes étapes de l'algorithme de Prim, en marquant initialement le sommet v_1 :

1)



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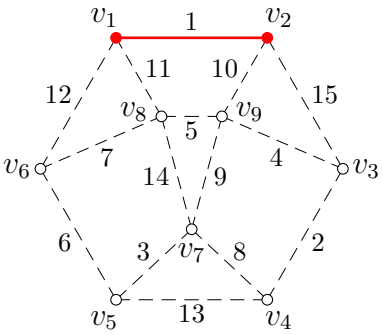
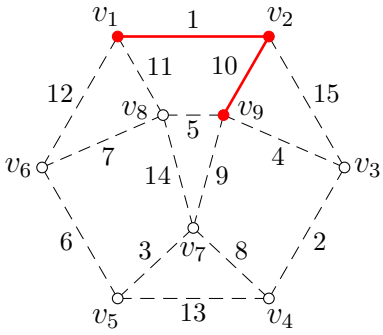
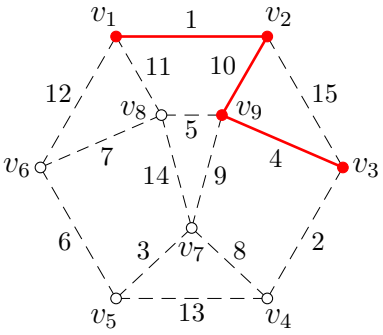


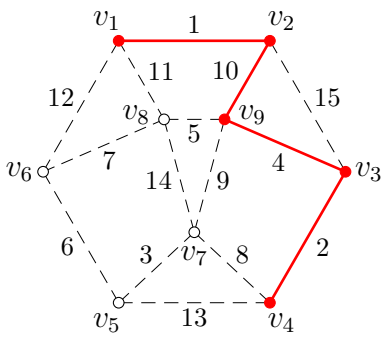
3)



4)



5)


v_1
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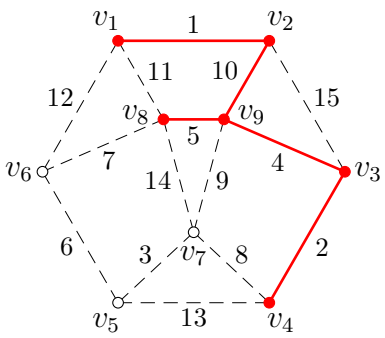
v_5
13
v_4

6)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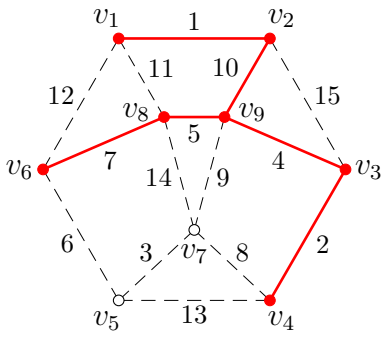
v_4

7)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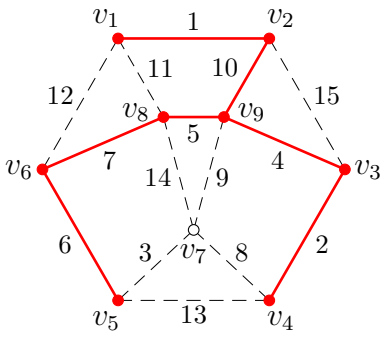
8)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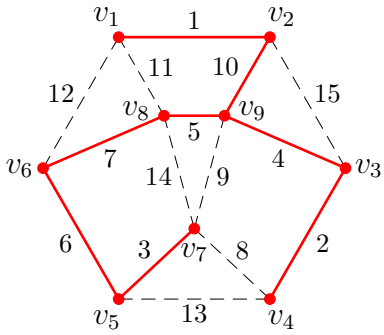
9)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