

1.4

$$1) \quad z \bar{z} = (a + bi)(a - bi) = a^2 - abi + abi - b^2 i^2 = a^2 + b^2$$

$$2) \quad \bar{\bar{z}} = \overline{a + bi} = \overline{a - bi} = a - (-b)i = a + bi = z$$

$$3) \quad \overline{z_1 + z_2} = \overline{(a_1 + b_1 i) + (a_2 + b_2 i)} = \overline{(a_1 + a_2) + (b_1 + b_2) i} = (a_1 + a_2) - (b_1 + b_2) i = (a_1 - b_1 i) + (a_2 - b_2 i) = \bar{z}_1 + \bar{z}_2$$

$$4) \quad \overline{z_1 - z_2} = \overline{(a_1 + b_1 i) - (a_2 + b_2 i)} = \overline{(a_1 - a_2) + (b_1 - b_2) i} = (a_1 - a_2) - (b_1 - b_2) i = (a_1 - b_1 i) - (a_2 - b_2 i) = \bar{z}_1 - \bar{z}_2$$

$$5) \quad \overline{z_1 z_2} = \overline{(a_1 + b_1 i)(a_2 + b_2 i)} = \overline{a_1 a_2 + a_1 b_2 i + a_2 b_1 i + b_1 b_2 i^2} = \overline{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i} = (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i$$

$$\bar{z}_1 \bar{z}_2 = \overline{(a_1 + b_1 i)} \overline{(a_2 + b_2 i)} = (a_1 - b_1 i)(a_2 - b_2 i) = a_1 a_2 - a_1 b_2 i - a_2 b_1 i + b_1 b_2 i^2 = (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i$$

$$\text{En définitive } \overline{z_1 z_2} = (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i = \bar{z}_1 \bar{z}_2$$

$$6) \quad \text{Utilisons la propriété } \overline{\bar{z}_1 z_2} = \overline{z_1 z_2} \text{ avec } z_1 = z \text{ et } z_2 = \frac{1}{z} :$$

$$\bar{z} \cdot \overline{\left(\frac{1}{z}\right)} = \overline{z \cdot \frac{1}{z}} = \overline{1} = 1$$

$$\text{En divisant cette égalité par } \bar{z}, \text{ on obtient : } \overline{\left(\frac{1}{z}\right)} = \frac{1}{\bar{z}}.$$

$$7) \quad \overline{\left(\frac{z_1}{z_2}\right)} = \overline{z_1 \left(\frac{1}{z_2}\right)} = \bar{z}_1 \overline{\left(\frac{1}{z_2}\right)} = \bar{z}_1 \frac{1}{\bar{z}_2} = \frac{\bar{z}_1}{\bar{z}_2}$$