

2.7 1) $z^4 + 1 = 0$ $S_{\mathbb{R}} = \emptyset$ car $z^4 \geq 0$ donc $z^4 + 1 \geq 1 \neq 0$
 $z^4 + 1 = 0 \Leftrightarrow z^4 = -1 \Leftrightarrow z^4 = e^{i\pi} \Leftrightarrow z = e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

2) $x^4 - x^2 - 1 = 0$ on pose $y = x^2 \Rightarrow y^2 - y - 1 = 0$ $\Delta = 1 + 4 = 5 \Rightarrow y_{1,2} = \frac{1 \pm \sqrt{5}}{2} = x_{1,2}^2$
 $S_{\mathbb{R}} = \left\{ \pm \sqrt{\frac{1 + \sqrt{5}}{2}} \right\}$ et $S_{\mathbb{C}} = \left\{ \pm \sqrt{\frac{1 + \sqrt{5}}{2}}; \pm \sqrt{\frac{1 - \sqrt{5}}{2}} i \right\}$

3) $P(x) = x^3 - 5x^2 + 4x - 20 = 0$ on essaie $\frac{P(20)}{P(1)} = \pm \{1; 2; 4; 5; 10; 20\}$

$P(1) \neq 0, P(2) \neq 0, P(-1) \neq 0, P(-2) \neq 0, P(4) \neq 0, P(-4) \neq 0, P(5) = 125 - 125 + 20 - 20 = 0$: racine

$\begin{array}{r|rrrr} 1 & 1 & -5 & 4 & -20 \\ & \downarrow & & & \\ 1 & 0 & -1 & 0 \end{array}$ donc $x_1 = 5$ et reste $x^2 + 4 = 0 \Rightarrow S_{\mathbb{R}} = \{5\}$ et $S_{\mathbb{C}} = \{5; \pm 2i\}$

4) $P(x) = x^3 + x - 2 = 0$ on essaie $\pm \{1; 2\}$ $P(1) = 0$: racine $\textcircled{1} \begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & \downarrow & & & \\ 1 & 0 & 1 & 0 \end{array}$ donc $x_1 = 1$
 et reste $x^2 + x + 2 = 0$ $\Delta = 1 - 8 = -7$ donc $S_{\mathbb{R}} = \{1\}$ et $S_{\mathbb{C}} = \left\{ 1; \frac{-1 \pm \sqrt{-7}}{2} \right\}$

5) $P(x) = x^3 + 1$ on essaie $P(-1) = 0$ $\textcircled{1} \begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & \downarrow & & & \\ -1 & 0 & 0 & 0 \end{array}$ donc $x_1 = -1$ et reste $x^2 + x + 1 = 0$
 $\Delta = 1 - 4 = -3$ donc $S_{\mathbb{R}} = \{-1\}$ et $S_{\mathbb{C}} = \left\{ -1; \frac{-1 \pm \sqrt{-3}}{2} \right\}$

6) $P(x) = x^5 - x^2 = x^2(x^3 - 1) = x^2(x - 1)(x^2 + x + 1)$ $\Delta = 1 - 4 = -3$ $S_{\mathbb{R}} = \{0; 1\}$ et $S_{\mathbb{C}} = \left\{ 0; 1; \frac{-1 \pm \sqrt{-3}}{2} \right\}$