3.2

1) $27\,q^2+4\,p^3=27\cdot(-26)^2+4\cdot 9^3=21\ 168>0 \qquad \frac{21\ 168}{27}=784=28^2$

$a=\sqrt[3]{-\frac{-26}{2}-\frac{1}{2}\cdot 28}=\sqrt[3]{-1}=-1$

$b=\sqrt[3]{-\frac{-26}{2}+\frac{1}{2}\cdot 28}=\sqrt[3]{27}=3$

$x_1=a+b=-1+3=2$

$x_2=a\,j+b\,j^2=-1\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\,i\right)+3\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\,i\right)=-1-2\sqrt{3}\,i$

$x_3=\overline{x_2}=-1+2\sqrt{3}\,i$

2) $27\,q^2+4\,p^3=27\cdot 2^2+4\cdot(-3)^3=0$

$a=\sqrt[3]{-\frac{2}{2}}=\sqrt[3]{-1}=-1$

$x_1=2\,a=2\cdot(-1)=-2$

$x_2=x_3=-a=-(-1)=1$

3) $27\,q^2+4\,p^3=27\cdot(-126)^2+4\cdot(-15)^3=415\ 152>0 \qquad \frac{415\ 152}{27}=15\ 376=124^2$

$a=\sqrt[3]{-\frac{-126}{2}-\frac{1}{2}\cdot 124}=\sqrt[3]{1}=1$

$b=\sqrt[3]{-\frac{-126}{2}+\frac{1}{2}\cdot 124}=\sqrt[3]{125}=5$

$x_1=a+b=1+5=6$

$x_2=a\,j+b\,j^2=1\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\,i\right)+5\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\,i\right)=-3-2\sqrt{3}\,i$

$x_3=\overline{x_2}=-3+2\sqrt{3}\,i$

4) $27\,q^2+4\,p^3=27\cdot(-56)^2+4\cdot 24^3=139\ 968>0 \qquad \frac{139\ 968}{27}=5184=72^2$

$a=\sqrt[3]{-\frac{-56}{2}-\frac{1}{2}\cdot 72}=\sqrt[3]{-8}=-2$

$b=\sqrt[3]{-\frac{-56}{2}+\frac{1}{2}\cdot 72}=\sqrt[3]{64}=4$

$x_1=a+b=-2+4=2$

$x_2=a\,j+b\,j^2=-2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\,i\right)+4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\,i\right)=-1-3\sqrt{3}\,i$

$x_3=\overline{x_2}=-1+3\sqrt{3}\,i$

5) $27\,q^2+4\,p^3=27\cdot(-16)^2+4\cdot(-12)^3=0$

$a=\sqrt[3]{-\frac{-16}{2}}=\sqrt[3]{8}=2$

$x_1=2\,a=2\cdot 2=4$

$x_2=x_3=-a=-2$

6) $27\,q^2+4\,p^3=27\cdot(-98)^2+4\cdot 45^3=623\ 808>0 \qquad \frac{623\ 808}{27}=23\ 104=152^2$

$a=\sqrt[3]{-\frac{-98}{2}-\frac{1}{2}\cdot 152}=\sqrt[3]{-27}=-3$

$b=\sqrt[3]{-\frac{-98}{2}+\frac{1}{2}\cdot 152}=\sqrt[3]{125}=5$

$x_1=a+b=-3+5=2$

$x_2 = a\,j + b\,j^2 = -3\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right) + 5\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\,i\right) = -1 - 4\sqrt{3}\,i$

$x_3 = \overline{x_2} = -1 + 4\sqrt{3}\,i$