

3.5

1) Si l'on divise l'équation $ax^3 + bx^2 + cx + d = 0$ par $a \neq 0$, on obtient :

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0.$$

2) Remplaçons x par $X - \frac{b}{3a}$:

$$\begin{aligned} & \left(X - \frac{b}{3a}\right)^3 + \frac{b}{a} \left(X - \frac{b}{3a}\right)^2 + \frac{c}{a} \left(X - \frac{b}{3a}\right) + \frac{d}{a} = \\ & X^3 - 3 \cdot \frac{b}{3a} X^2 + 3 \cdot \left(\frac{b}{3a}\right)^2 X - \left(\frac{b}{3a}\right)^3 + \frac{b}{a} \left(X^2 - 2 \cdot \frac{b}{3a} X + \left(\frac{b}{3a}\right)^2\right) + \\ & \frac{c}{a} \left(X - \frac{b}{3a}\right) + \frac{d}{a} = \\ & X^3 - \frac{b}{a} X^2 + \frac{b^2}{3a^2} X - \frac{b^3}{27a^3} + \frac{b}{a} X^2 - \frac{2b^2}{3a^2} X + \frac{b^3}{9a^3} + \frac{c}{a} X - \frac{bc}{3a^2} + \frac{d}{a} = \\ & X^3 + \underbrace{\left(\frac{c}{a} - \frac{b^2}{3a^2}\right)}_p X + \underbrace{\left(\frac{2b^3}{27a^3} - \frac{bc}{3a^2} + \frac{d}{a}\right)}_q = \\ & X^3 + pX + q = 0 \end{aligned}$$