

4.6

- 1) $|2 + 2i| = |2(1 + i)| = |2||1 + i| = 2\sqrt{1^2 + 1^2} = 2\sqrt{2}$
 $2 + 2i = 2\sqrt{2} \left(\frac{2}{2\sqrt{2}} + i \frac{2}{2\sqrt{2}} \right) = 2\sqrt{2} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = 2\sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right)$
 $r = 2\sqrt{2} \quad \varphi = \frac{\pi}{4}$
- 2) $|3\sqrt{3} + 3i| = |3(\sqrt{3} + i)| = |3||\sqrt{3} + i| = 3\sqrt{(\sqrt{3})^2 + 1^2} = 3\sqrt{4} = 3 \cdot 2 = 6$
 $3\sqrt{3} + 3i = 6 \left(\frac{3\sqrt{3}}{6} + i \frac{3}{6} \right) = 6 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right)$
 $r = 6 \quad \varphi = \frac{\pi}{6}$
- 3) $|1 - \sqrt{3}i| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2$
 $1 - \sqrt{3}i = 2 \left(\frac{1}{2} + i \left(-\frac{\sqrt{3}}{2} \right) \right)$
 $r = 2 \quad \varphi = \frac{5\pi}{3}$
- 4) $|5i| = |5||i| = 5\sqrt{0^2 + 1^2} = 5\sqrt{1} = 5$
 $5i = 5(0 + i \cdot 1)$
 $r = 5 \quad \varphi = \frac{\pi}{2}$
- 5) $|-3| = 3$
 $-3 = 3(-1 + i \cdot 0)$
 $r = 3 \quad \varphi = \pi$
- 6) $|-2\sqrt{3} - 2i| = |-2(\sqrt{3} + i)| = |-2||\sqrt{3} + i| = 2\sqrt{(\sqrt{3})^2 + 1^2} = 2\sqrt{3 + 1} = 2\sqrt{4} = 2 \cdot 2 = 4$
 $-2\sqrt{3} - 2i = 4 \left(-\frac{2\sqrt{3}}{4} + i \left(-\frac{2}{4} \right) \right) = 4 \left(-\frac{\sqrt{3}}{2} + i \left(-\frac{1}{2} \right) \right)$
 $r = 4 \quad \varphi = \frac{7\pi}{6}$
- 7) $|-7 - 7i| = |-7(1 + i)| = |-7||1 + i| = 7\sqrt{1^2 + 1^2} = 7\sqrt{2}$
 $-7 - 7i = 7\sqrt{2} \left(-\frac{7}{7\sqrt{2}} + i \left(-\frac{7}{7\sqrt{2}} \right) \right) = 7\sqrt{2} \left(-\frac{1}{\sqrt{2}} + i \left(-\frac{1}{\sqrt{2}} \right) \right) = 7\sqrt{2} \left(-\frac{\sqrt{2}}{2} + i \left(-\frac{\sqrt{2}}{2} \right) \right)$
 $r = 7\sqrt{2} \quad \varphi = \frac{5\pi}{4}$
- 8) $|-3i| = |-3||i| = 3\sqrt{0^2 + 1^2} = 3\sqrt{1} = 3$
 $-3i = 3(0 + i \cdot (-1))$
 $r = 3 \quad \varphi = \frac{3\pi}{2}$
- 9) $|\sin(\alpha) + i \cos(\alpha)| = \sqrt{\sin^2(\alpha) + \cos^2(\alpha)} = \sqrt{1} = 1$
 $\sin(\alpha) + i \cos(\alpha) = \cos\left(\frac{\pi}{2} - \alpha\right) + i \sin\left(\frac{\pi}{2} - \alpha\right)$
 $r = 1 \quad \varphi = \frac{\pi}{2} - \alpha$