

11.18 1) $\cos(x) = \sin(x)$

$$\sin\left(\frac{\pi}{2} - x\right) = \sin(x)$$

$$\begin{cases} \frac{\pi}{2} - x_1 = x_1 + 2k\pi & \text{où } k \in \mathbb{Z} \\ \frac{\pi}{2} - x_2 = \pi - x_2 + 2k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} -2x_1 = -\frac{\pi}{2} + 2k\pi & \text{où } k \in \mathbb{Z} \\ 0 = \frac{\pi}{2} + 2k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} x_1 = \frac{\pi}{4} - k\pi & \text{où } k \in \mathbb{Z} \\ \text{impossible} \end{cases}$$

2) $\cos(x) = -\sin\left(\frac{x}{2}\right)$

$$\cos(x) = \sin\left(-\frac{x}{2}\right)$$

$$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(-\frac{x}{2}\right)$$

$$\begin{cases} \frac{\pi}{2} - x_1 = -\frac{x_1}{2} + 2k\pi & \text{où } k \in \mathbb{Z} \\ \frac{\pi}{2} - x_2 = \pi - \left(-\frac{x_2}{2}\right) + 2k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} \pi - 2x_1 = -x_1 + 4k\pi & \text{où } k \in \mathbb{Z} \\ \pi - 2x_2 = 2\pi + x_2 + 4k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} -x_1 = -\pi + 4k\pi & \text{où } k \in \mathbb{Z} \\ -3x_2 = \pi + 4k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} x_1 = \pi - 4k\pi & \text{où } k \in \mathbb{Z} \\ x_2 = -\frac{\pi}{3} - \frac{4k\pi}{3} & \text{où } k \in \mathbb{Z} \end{cases}$$

3) $\sin(2x) = \cos\left(\frac{\pi}{3} - x\right)$

$$\cos\left(\frac{\pi}{2} - 2x\right) = \cos\left(\frac{\pi}{3} - x\right)$$

$$\begin{cases} \frac{\pi}{2} - 2x_1 = \frac{\pi}{3} - x_1 + 2k\pi & \text{où } k \in \mathbb{Z} \\ \frac{\pi}{2} - 2x_2 = -\left(\frac{\pi}{3} - x_2\right) + 2k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

En multipliant ces équations par 6, on obtient :

$$\begin{cases} 3\pi - 12x_1 = 2\pi - 6x_1 + 12k\pi & \text{où } k \in \mathbb{Z} \\ 3\pi - 12x_2 = -2\pi + 6x_2 + 12k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} -6x_1 = -\pi + 12k\pi & \text{où } k \in \mathbb{Z} \\ -18x_2 = -5\pi + 12k\pi & \text{où } k \in \mathbb{Z} \end{cases}$$

$$\begin{cases} x_1 = \frac{\pi}{6} - 2k\pi & \text{où } k \in \mathbb{Z} \\ x_2 = \frac{5\pi}{18} - \frac{2k\pi}{3} & \text{où } k \in \mathbb{Z} \end{cases}$$