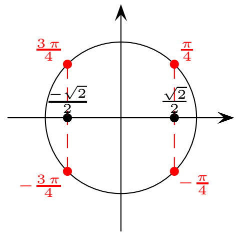
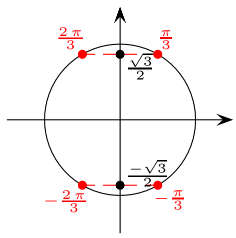
11.19 1) $\cos^2(x) = \frac{1}{2}$



$\cos(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$$\begin{cases} x_1 = \frac{\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_2 = -\frac{\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_3 = \frac{3\,\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_4 = -\frac{3\,\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \end{cases}$$

2) $\sin^2(x) = \frac{3}{4}$



$\sin(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$

$$\begin{cases} x_1 = \frac{\pi}{3} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_2 = \frac{2\,\pi}{3} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_3 = -\frac{2\,\pi}{3} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_4 = -\frac{\pi}{3} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \end{cases}$$

3) $\cos^2(x) = \sin^2(x)$

En posant $a = \cos(x)$ et $b = \sin(x)$, il s'agit de résoudre le système :

$$\begin{cases} a^2 = b^2 \\ a^2 + b^2 = 1 \end{cases}$$

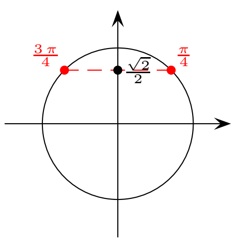
$$\begin{cases} a^2 = b^2 \\ b^2 + b^2 = 1 \end{cases}$$

$$\begin{cases} a^2 = b^2 \\ 2\,b^2 - 1 = 0 \end{cases}$$

$$\begin{cases} a^2 = b^2 \\ (\sqrt{2}\,b - 1)\,(\sqrt{2}\,b + 1) = 0 \end{cases}$$

$$\begin{cases} a = \pm b \\ \sqrt{2}\,b - 1 = 0 \end{cases} \quad \text{ou} \quad \begin{cases} a = \pm b \\ \sqrt{2}\,b + 1 = 0 \end{cases}$$

$$\begin{cases} a = \pm\frac{\sqrt{2}}{2} \\ b = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases} \quad \text{ou} \quad \begin{cases} a = \pm\frac{\sqrt{2}}{2} \\ b = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{cases}$$



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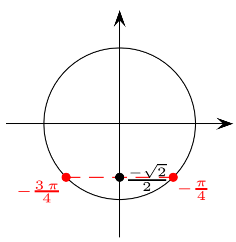


$$\begin{cases} x_1 = \frac{\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_2 = \frac{3\,\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_3 = -\frac{\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_4 = -\frac{3\,\pi}{4} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \end{cases}$$

4) $2\,\cos^2(x) - \sin(x) - 1 = 0$

En posant $a = \cos(x)$ et $b = \sin(x)$, il s'agit de résoudre le système :

$$\begin{cases} 2\,a^2 - b - 1 = 0 \\ a^2 + b^2 = 1 \end{cases}$$

$$\begin{cases} 2\,a^2 - b - 1 = 0 \\ a^2 = 1 - b^2 \end{cases}$$

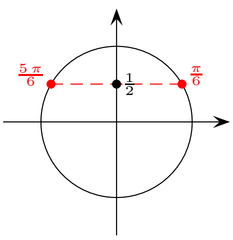
$$\begin{cases} 2\,(1 - b^2) - b - 1 = 0 \\ a^2 = 1 - b^2 \end{cases}$$

$$\begin{cases} 0 = 2\,b^2 + b - 1 \\ a^2 = 1 - b^2 \end{cases}$$

Déterminons les solutions de la première équation à l'aide du discriminant :
$\Delta = 1^2 - 4 \cdot 2 \cdot (-1) = 9$

$$\begin{cases} b = \frac{-1+\sqrt{9}}{2 \cdot 2} = \frac{1}{2} \\ a^2 = 1 - (\frac{1}{2})^2 = 1 - \frac{1}{4} = \frac{3}{4} \end{cases} \quad \text{ou} \quad \begin{cases} b = \frac{-1-\sqrt{9}}{2 \cdot 2} = -1 \\ a^2 = 1 - (-1)^2 = 1 - 1 = 0 \end{cases}$$

$$\begin{cases} b = \frac{1}{2} \\ a = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2} \end{cases} \quad \text{ou} \quad \begin{cases} b = -1 \\ a = 0 \end{cases}$$



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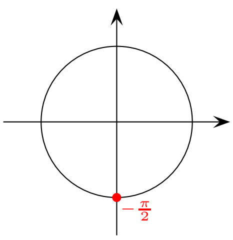

$$\begin{cases} x_1 = \frac{\pi}{6} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_2 = \frac{5\,\pi}{6} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \\ x_3 = -\frac{\pi}{2} + 2\,k\,\pi \text{ où } k \in \mathbb{Z} \end{cases}$$