

2.12

1) $(x+1)(x^2-4) = 3(x-2)(x+1)$

$$(x+1)(x^2-4) - 3(x-2)(x+1) = 0$$

$$(x+1)(x-2)(x+2) - 3(x-2)(x+1) = 0$$

$$(x+1)(x-2)((x+2)-3) = 0$$

$$(x+1)(x-2)(x-1) = 0$$

$$x+1=0 \quad \text{ou} \quad x-2=0 \quad \text{ou} \quad x-1=0$$

$$x=-1 \quad \text{ou} \quad x=2 \quad \text{ou} \quad x=1$$

$$S = \{-1; 1; 2\}$$

2) $x^3 + x^2 = 4x + 4$

$$x^3 + x^2 - 4x - 4 = 0$$

$$x^2(x+1) - 4(x+1) = 0$$

$$(x+1)(x^2-4) = 0$$

$$(x+1)(x-2)(x+2) = 0$$

$$x+1=0 \quad \text{ou} \quad x-2=0 \quad \text{ou} \quad x+2=0$$

$$x=-1 \quad \text{ou} \quad x=2 \quad \text{ou} \quad x=-2$$

$$S = \{-2; -1; 2\}$$

3) $x^4 + 4 = 5x^2$

$$x^4 - 5x^2 + 4 = 0$$

$$(x^2-1)(x^2-4) = 0$$

$$(x-1)(x+1)(x-2)(x+2) = 0$$

$$x-1=0 \quad \text{ou} \quad x+1=0 \quad \text{ou} \quad x-2=0 \quad \text{ou} \quad x+2=0$$

$$x=1 \quad \text{ou} \quad x=-1 \quad \text{ou} \quad x=2 \quad \text{ou} \quad x=-2$$

$$S = \{-2; -1; 1; 2\}$$

4) $9x^2 - (2x+5)^2 = 0$

$$(3x - (2x+5))(3x + (2x+5)) = 0$$

$$(x-5)(5x+5) = 5(x-5)(x+1) = 0$$

$$x-5=0 \quad \text{ou} \quad x+1=0$$

$$x=5 \quad \text{ou} \quad x=-1$$

$$S = \{-1; 5\}$$

5) $2(x^2-9) = x+3$

$$2(x^2-9) - (x+3) = 0$$

$$2(x-3)(x+3) - (x+3) = 0$$

$$(x+3)(2(x-3)-1)=0$$

$$(x+3)(2x-7)=0$$

$$x+3=0 \quad \text{ou} \quad 2x-7=0$$

$$x=-3 \quad \text{ou} \quad x=\frac{7}{2}$$

$$S=\left\{-3;\frac{7}{2}\right\}$$

$$6) \quad x^3 = x^2$$

$$x^3 - x^2 = 0$$

$$x^2(x-1)=0$$

$$x=0 \quad \text{ou} \quad x-1=0$$

$$x=0 \text{ (solution double)} \quad \text{ou} \quad x=1$$

$$S=\{0;1\}$$

$$7) \quad x^3 + 2x^2 - x - 2 = 0$$

$$x^2(x+2)-(x+2)=0$$

$$(x+2)(x^2-1)=0$$

$$(x+2)(x-1)(x+1)=0$$

$$x+2=0 \quad \text{ou} \quad x-1=0 \quad \text{ou} \quad x+1=0$$

$$x=-2 \quad \text{ou} \quad x=1 \quad \text{ou} \quad x=-1$$

$$S=\{-2;-1;1\}$$

$$8) \quad x^3 - 3x^2 + 3x - 1 = 0$$

$$(x-1)^3=0$$

$$x-1=0$$

$$x=1 \text{ (solution triple)}$$

$$S=\{1\}$$

$$9) \quad x^5 - 2x^4 = x - 2$$

$$x^5 - 2x^4 - x + 2 = 0$$

$$x^4(x-2)-(x-2)=0$$

$$(x-2)(x^4-1)=0$$

$$(x-2)(x^2-1)(x^2+1)$$

$$(x-2)(x-1)(x+1)(x^2+1)=0$$

$$x-2=0 \quad \text{ou} \quad x-1=0 \quad \text{ou} \quad x+1=0 \quad \text{ou} \quad x^2+1=0$$

$$x=2 \quad \text{ou} \quad x=1 \quad \text{ou} \quad x=-1$$

L'équation $x^2+1=0$ n'a aucune solution réelle : pour tout $x \in \mathbb{R}$, on a $x^2 \geq 0$, c'est-à-dire $x^2+1 \geq 1 > 0$.

$$S = \{-1; 1; 2\}$$

$$10) \quad x^3 - 2x^2 - 3x = 0$$

$$x(x^2 - 2x - 3) = 0$$

$$x(x - 3)(x + 1) = 0$$

$$x = 0 \quad \text{ou} \quad x - 3 = 0 \quad \text{ou} \quad x + 1 = 0$$

$$x = 0 \quad \text{ou} \quad x = 3 \quad \text{ou} \quad x = -1$$

$$S = \{-1; 0; 3\}$$

$$11) \quad x^8 + 9x^4 - 10 = 0$$

$$(x^4 - 1)(x^4 + 10) = 0$$

$$(x^2 - 1)(x^2 + 1)(x^4 + 10) = 0$$

$$(x - 1)(x + 1)(x^2 + 1)(x^4 + 10) = 0$$

$$x - 1 = 0 \quad \text{ou} \quad x + 1 = 0 \quad \text{ou} \quad x^2 + 1 = 0 \quad \text{ou} \quad x^4 + 10 = 0$$

$$x = 1 \quad \text{ou} \quad x = -1$$

L'équation $x^2 + 1 = 0$ n'a aucune solution réelle : pour tout $x \in \mathbb{R}$, on a $x^2 \geq 0$, c'est-à-dire $x^2 + 1 \geq 1 > 0$.

L'équation $x^4 + 10 = 0$ n'a aucune solution réelle : pour tout $x \in \mathbb{R}$, on a $x^4 = (x^2)^2 \geq 0$, c'est-à-dire $x^4 + 10 \geq 10 > 0$.

$$S = \{-1; 1\}$$

$$12) \quad 4x^3 - 4x^2 + x = 0$$

$$x(4x^2 - 4x + 1) = 0$$

$$x(2x - 1)^2 = 0$$

$$x = 0 \quad \text{ou} \quad 2x - 1 = 0$$

$$x = 0 \quad \text{ou} \quad x = \frac{1}{2} \text{ (solution double)}$$

$$S = \left\{0; \frac{1}{2}\right\}$$

$$13) \quad 4x^3 = 36x$$

$$4x^3 - 36x = 0$$

$$4x(x^2 - 9) = 0$$

$$4x(x - 3)(x + 3) = 0$$

$$4x = 0 \quad \text{ou} \quad x - 3 = 0 \quad \text{ou} \quad x + 3 = 0$$

$$x = 0 \quad \text{ou} \quad x = 3 \quad \text{ou} \quad x = -3$$

$$S = \{-3; 0; 3\}$$

$$14) \quad x^4 + 15x^2 - 16 = 0$$

$$(x^2 - 1)(x^2 + 16) = 0$$

$$(x - 1)(x + 1)(x^2 + 16) = 0$$

$$x - 1 = 0 \quad \text{ou} \quad x + 1 = 0 \quad \text{ou} \quad x^2 + 16 = 0$$

$$x = 1 \quad \text{ou} \quad x = -1$$

L'équation $x^2 + 16 = 0$ n'admet aucune solution réelle : quel que soit $x \in \mathbb{R}$, on constate que $x^2 \geq 0$, d'où suit $x^2 + 16 \geq 16 > 0$.

$$S = \{-1; 1\}$$

$$15) \quad x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x - 1 = 0$$

$$x = 1 \quad (\text{solution double})$$

$$S = \{1\}$$

$$16) \quad 4(x - 3) = x^2 - 9$$

$$4(x - 3) = (x - 3)(x + 3)$$

$$4(x - 3) - (x - 3)(x + 3) = 0$$

$$(x - 3)(4 - (x + 3)) = 0$$

$$(x - 3)(1 - x) = 0$$

$$x - 3 = 0 \quad \text{ou} \quad 1 - x = 0$$

$$x = 3 \quad \text{ou} \quad x = 1$$

$$S = \{1; 3\}$$

$$17) \quad 3x^3 + 2x^2 - 3x - 2 = 0$$

$$x^2(3x + 2) - 1(3x + 2) = 0$$

$$(3x + 2)(x^2 - 1) = 0$$

$$(3x + 2)(x - 1)(x + 1) = 0$$

$$3x + 2 = 0 \quad \text{ou} \quad x - 1 = 0 \quad \text{ou} \quad x + 1 = 0$$

$$x = -\frac{2}{3} \quad \text{ou} \quad x = 1 \quad \text{ou} \quad x = -1$$

$$S = \left\{-1; -\frac{2}{3}; 1\right\}$$

$$18) \quad 3(x^3 + 8) = (x + 2)^3$$

$$3(x + 2)(x^2 - 2x + 4) = (x + 2)^3$$

$$3(x + 2)(x^2 - 2x + 4) - (x + 2)^3 = 0$$

$$(x + 2)(3(x^2 - 2x + 4) - (x + 2)^2) = 0$$

$$(x+2)(3x^2-6x+12-x^2-4x-4)=0$$

$$(x+2)(2x^2-10x+8)=0$$

$$(x+2)2(x^2-5x+4)=0$$

$$2(x+2)(x-1)(x-4)=0$$

$$x+2=0 \quad \text{ou} \quad x-1=0 \quad \text{ou} \quad x-4=0$$

$$x=-2 \quad \text{ou} \quad x=1 \quad \text{ou} \quad x=4$$

$$S = \{-2; 1; 4\}$$

$$19) \quad 2x(x-1) = x(2x+7)$$

$$2x(x-1) - x(2x+7) = 0$$

$$x(2(x-1) - (2x+7)) = 0$$

$$x(-9) = -9x = 0$$

$$x = 0$$

$$S = \{0\}$$

$$20) \quad (2x^2+3x+1)^2 - (2x^2-4x-1)^2 = 0$$

$$\left((2x^2+3x+1) - (2x^2-4x-1)\right) \left((2x^2+3x+1) + (2x^2-4x-1)\right) = 0$$

$$(7x+2)(4x^2-x) = 0$$

$$(7x+2)x(4x-1) = 0$$

$$7x+2=0 \quad \text{ou} \quad x=0 \quad \text{ou} \quad 4x-1=0$$

$$x = -\frac{2}{7} \quad \text{ou} \quad x=0 \quad \text{ou} \quad x = \frac{1}{4}$$

$$S = \left\{-\frac{2}{7}; 0; \frac{1}{4}\right\}$$