

4 Fractions rationnelles

4.1 Simplifier les fractions rationnelles suivantes :

$$\begin{array}{lll} 1) \frac{x^2 - 8x + 15}{x^2 - 7x + 12} & 2) \frac{6x^2 + 2x}{27x^3 + 1} & 3) \frac{6x^2 + 11x + 6}{2x + 3} \\ 4) \frac{x^2 - 25}{x^2 + 10x + 25} & 5) \frac{x - a}{ax - a^2} & 6) \frac{x^3 + x^2 + x + 1}{x^3 - x^2 + x - 1} \\ 7) \frac{x^3 - 1}{x^2 - 1} & 8) \frac{x^4 - 4}{x^2 - 2} & 9) \frac{x^3 - 27}{3x^2 - 3x - 18} \end{array}$$

4.2 Simplifier.

$$\begin{array}{lll} 1) \frac{x^3 - 4x}{x^4 - x^3 - 2x^2} & 2) \frac{7x^3 - 3x^2 + 7x - 3}{7x - 3} & 3) \frac{x - 3}{4x^3 - 14x^2 + 18} \\ 4) \frac{x^3 - x}{x^2 + x} & 5) \frac{x^3 + 6x^2 + 15x + 18}{x^2 - 2x - 15} & 6) \frac{x^3 - x^2 - 17x - 15}{x^2 - 1} \\ 7) \frac{x^2 + x + 1}{x^2 - x + 1} & 8) \frac{x^2 - 10x + 21}{x^2 - 14x + 49} & 9) \frac{x^2 - 6x + 8}{x^3 - 5x^2 + 2x + 8} \end{array}$$

4.3 Simplifier et effectuer.

$$\begin{array}{ll} 1) \frac{x^2 - 1}{x^2 + 6x + 9} \cdot \frac{x^2 - 9}{x + 1} & 2) \frac{x^2 + ax}{x - a} \cdot \frac{ax - x^2}{x^3} \\ 3) \frac{x^2 + 1}{x + 1} : \frac{x^4 - 1}{x^2 - 5x + 6} & 4) \frac{(x + 4)(x - 3)}{x^2 - 3x} : \frac{x^2 - 6x + 9}{2x^2 - 8x + 6} \\ 5) \frac{x^2 + xy}{x - y} \cdot \frac{xy - x^2}{x^3} & 6) \frac{x^2 - xy}{x^2 + xy} \cdot \frac{x^2 - y^2}{3xy^2} \cdot \frac{x^2 y^2}{x^2 - 2xy + y^2} \end{array}$$

4.4 Simplifier et effectuer.

$$\begin{array}{ll} 1) \frac{x + 4}{x^2 + x - 12} \cdot \frac{x^2 - x - 6}{2x + 4} & 2) \frac{2x^2 + 3x - 2}{x^2 - 2x + 1} \cdot \frac{x - 1}{4x^2 - 16} \\ 3) \frac{x^3 - 3x^2 + 4x - 12}{x - 2} : \frac{x - 3}{x^2 - 4} & 4) (1 + x) : \frac{x^2}{x^2 - 6x - 7} \\ 5) \frac{x^2 - xy}{xy + y^2} : \frac{xy - y^2}{x^2 + xy} & 6) \frac{xy - x + y - 1}{3x^2 + 3x} : \frac{xy - x + 2y - 2}{4x^2 + 8x} \end{array}$$

4.5 Effectuer et simplifier.

- 1) $\frac{x}{x+y} + \frac{y}{x-y} - \frac{2xy}{x^2-y^2}$
- 2) $\frac{2x+y}{3x^2} - \frac{3x+y}{2xy} + \frac{2x+y}{6y^2} - \frac{y-8x}{6xy}$
- 3) $\frac{x+y}{x-y} + \frac{x-2y}{x+y} + \frac{x^2+3y^2}{y^2-x^2}$
- 4) $\frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{x-x^3}$
- 5) $\frac{4}{x^2-y^2} + \frac{3y}{x^2y-x^3} - \frac{x-3y}{x^3-xy^2}$
- 6) $\frac{x^3-y^3}{x^2-y^2} - \frac{x^2y+xy^2}{x^2+xy}$

4.6 Effectuer et simplifier.

- 1) $\frac{3-2x}{2x+3} - \frac{2x+3}{3-2x} + \frac{36}{4x^2-9}$
- 2) $\frac{x-3}{x+3} - \frac{4x-6y}{xy+3y+2x+6} + \frac{y+6}{y+2}$
- 3) $\frac{1}{(x-y)(x-z)} + \frac{1}{(y-x)(y-z)} + \frac{1}{(z-x)(z-y)}$
- 4) $\frac{y-z}{(x-y)(x-z)} - \frac{z-x}{(y-x)(y-z)} + \frac{x-y}{(z-x)(z-y)}$
- 5) $\frac{x-3}{x^2+6x+8} - \frac{x-2}{x^2+7x+12} + \frac{5}{x^2+5x+6}$
- 6) $\frac{2x^2-x-7}{x^4-5x^2+4} - \frac{x-3}{x^3-2x^2-x+2}$

4.7 Effectuer et simplifier.

- 1) $\left(\frac{x+2}{x} - \frac{2}{x^2+x}\right) \left(\frac{1}{x} + 1\right)$
- 2) $\left[\left(x + \frac{2x}{x-2}\right) \left(\frac{2x}{x-2} - 2\right)\right] : \frac{4x^2}{x^2-4}$
- 3) $\left(\frac{1}{x} - \frac{1}{x^2}\right) : \left[\left(\frac{y}{x} + 1\right) \left(\frac{y}{x} - 1\right) \frac{x}{y^2-x^2} - \frac{1}{x^2}\right]$
- 4) $\left(\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}\right) : \left(\frac{1}{x^2} - 1\right)$
- 5) $\frac{2x-1}{x+y} \cdot \frac{x^2-y^2}{4x^2-1} \cdot \left(1 - \frac{2x}{2x-1} + \frac{1}{2x+1}\right)$
- 6) $\left(\frac{x}{x+y} + \frac{y}{x-y} - 1\right) \left(\frac{y}{x} - \frac{x}{y}\right) - \left(1 - \frac{x}{y}\right) \left(2 - \frac{2y}{x}\right)$

Réponses

- 4.1**
- | | | |
|--------------------------|---------------------------|------------------------------|
| 1) $\frac{x-5}{x-4}$ | 2) $\frac{2x}{9x^2-3x+1}$ | 3) $\frac{6x^2+11x+6}{2x+3}$ |
| 4) $\frac{x-5}{x+5}$ | 5) $\frac{1}{a}$ | 6) $\frac{x+1}{x-1}$ |
| 7) $\frac{x^2+x+1}{x+1}$ | 8) x^2+2 | 9) $\frac{x^2+3x+9}{3(x+2)}$ |
- 4.2**
- | | | |
|------------------------------|---------------------------|-----------------------------|
| 1) $\frac{x+2}{x(x+1)}$ | 2) x^2+1 | 3) $\frac{1}{2(x+1)(2x-3)}$ |
| 4) $x-1$ | 5) $\frac{x^2+3x+6}{x-5}$ | 6) $\frac{(x-5)(x+3)}{x-1}$ |
| 7) $\frac{x^2+x+1}{x^2-x+1}$ | 8) $\frac{x-3}{x-7}$ | 9) $\frac{1}{x+1}$ |
- 4.3**
- | | | |
|---------------------------------|---------------------|--------------------------------------|
| 1) $\frac{(x-3)(x-1)}{x+3}$ | 2) $-\frac{x+a}{x}$ | 3) $\frac{(x-2)(x-3)}{(x-1)(x+1)^2}$ |
| 4) $\frac{2(x-1)(x+4)}{x(x-3)}$ | 5) $-\frac{x+y}{x}$ | 6) $\frac{x}{3}$ |
- 4.4**
- | | | |
|-------------------------------|-------------------------------|-------------------|
| 1) $\frac{1}{2}$ | 2) $\frac{2x-1}{4(x-1)(x-2)}$ | 3) $(x^2+4)(x+2)$ |
| 4) $\frac{(x+1)^2(x-7)}{x^2}$ | 5) $\frac{x^2}{y^2}$ | 6) $\frac{4}{3}$ |
- 4.5**
- | | | |
|---------------------------|------------------------------|----------------------|
| 1) $\frac{x-y}{x+y}$ | 2) $\frac{x^3+y^3}{3x^2y^2}$ | 3) $\frac{x}{x+y}$ |
| 4) $\frac{2}{(x+1)(x-1)}$ | 5) $\frac{3}{x^2}$ | 6) $\frac{x^2}{x+y}$ |
- 4.6**
- | | | |
|----------------------|---------------------------|---------------------------|
| 1) $\frac{12}{2x-3}$ | 2) 2 | 3) 0 |
| 4) $\frac{2}{z-x}$ | 5) $\frac{5}{(x+2)(x+4)}$ | 6) $\frac{1}{(x+2)(x-2)}$ |
- 4.7**
- | | | |
|-------------------------|------------------------------------|------------------------|
| 1) $\frac{x+3}{x}$ | 2) $\frac{x+2}{x-2}$ | 3) 1 |
| 4) $\frac{x-3}{x(1-x)}$ | 5) $\frac{2(y-x)}{(2x+1)^2(2x-1)}$ | 6) $\frac{2(x-2y)}{y}$ |