

$$\begin{aligned}
1) \quad \frac{x}{x+y} + \frac{y}{x-y} - \frac{2xy}{x^2-y^2} &= \frac{x}{x+y} + \frac{y}{x-y} - \frac{2xy}{(x-y)(x+y)} = \\
&= \frac{x(x-y) + y(x+y) - 2xy}{(x-y)(x+y)} \\
&= \frac{x^2 - xy + xy + y^2 - 2xy}{(x-y)(x+y)} \\
&= \frac{x^2 - 2xy + y^2}{(x-y)(x+y)} = \frac{(x-y)^2}{(x-y)(x+y)} \\
&= \frac{x-y}{x+y}
\end{aligned}$$

$$\begin{aligned}
2) \quad \frac{2x+y}{3x^2} - \frac{3x+y}{2xy} + \frac{2x+y}{6y^2} - \frac{y-8x}{6xy} \\
&= \frac{(2x+y)2y^2 - (3x+y)3xy + (2x+y)x^2 - (y-8x)xy}{6x^2y^2} \\
&= \frac{4xy^2 + 2y^3 - 9x^2y - 3xy^2 + 2x^3 + x^2y - xy^2 + 8x^2y}{6x^2y^2} \\
&= \frac{2x^3 + 2y^3}{6x^2y^2} = \frac{2(x^3 + y^3)}{6x^2y^2} = \frac{x^3 + y^3}{3x^2y^2}
\end{aligned}$$

$$\begin{aligned}
3) \quad \frac{x+y}{x-y} + \frac{x-2y}{x+y} + \frac{x^2+3y^2}{y^2-x^2} &= \frac{x+y}{x-y} + \frac{x-2y}{x+y} - \frac{x^2+3y^2}{x^2-y^2} \\
&= \frac{x+y}{x-y} + \frac{x-2y}{x+y} - \frac{x^2+3y^2}{(x-y)(x+y)} \\
&= \frac{(x+y)^2 + (x-2y)(x-y) - (x^2+3y^2)}{(x-y)(x+y)} \\
&= \frac{x^2 + 2xy + y^2 + x^2 - xy - 2xy + 2y^2 - x^2 - 3y^2}{(x-y)(x+y)} \\
&= \frac{x^2 - xy}{(x-y)(x+y)} = \frac{x(x-y)}{(x-y)(x+y)} = \frac{x}{x+y}
\end{aligned}$$

$$\begin{aligned}
4) \quad \frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{x-x^3} &= \frac{1}{x} - \frac{x}{(x-1)(x+1)} - \frac{2x+1}{x(1-x^2)} \\
&= \frac{1}{x} - \frac{x}{(x-1)(x+1)} + \frac{2x+1}{x \underbrace{(x^2-1)}_{(x-1)(x+1)}} \\
&= \frac{(x-1)(x+1) - x^2 + (2x+1)}{x(x-1)(x+1)} \\
&= \frac{x^2 - 1 - x^2 + 2x + 1}{x(x-1)(x+1)} = \frac{2x}{x(x-1)(x+1)}
\end{aligned}$$

$$= \frac{2}{(x-1)(x+1)}$$

$$\begin{aligned} 5) \quad & \frac{4}{x^2 - y^2} + \frac{3y}{x^2 y - x^3} - \frac{x - 3y}{x^3 - x y^2} = \frac{4}{(x-y)(x+y)} + \frac{3y}{x^2(y-x)} - \frac{x-3y}{x(x^2-y^2)} \\ &= \frac{4}{(x-y)(x+y)} - \frac{3y}{x^2(x-y)} - \frac{x-3y}{x(x-y)(x+y)} \\ &= \frac{4x^2 - 3y(x+y) - (x-3y)x}{x^2(x-y)(x+y)} = \frac{4x^2 - 3xy - 3y^2 - x^2 + 3xy}{x^2(x-y)(x+y)} \\ &= \frac{3(x^2 - y^2)}{x^2(x-y)(x+y)} = \frac{3(x-y)(x+y)}{x^2(x-y)(x+y)} = \frac{3}{x^2} \end{aligned}$$

$$\begin{aligned} 6) \quad & \frac{x^3 - y^3}{x^2 - y^2} - \frac{x^2 y + x y^2}{x^2 + x y} = \frac{(x-y)(x^2 + x y + y^2)}{(x-y)(x+y)} - \frac{x y (x+y)}{x(x+y)} \\ &= \frac{x^2 + x y + y^2}{x+y} - y = \frac{(x^2 + x y + y^2) - y(x+y)}{x+y} \\ &= \frac{x^2 + x y + y^2 - x y - y^2}{x+y} = \frac{x^2}{x+y} \end{aligned}$$