

5 Étude du signe & Inéquations

5.1 Étudier le signe des polynômes suivants :

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|-------------|------------------------|---------------------------------|
| 1) $2x + 1$ | 2) $-3x + 7$ | 3) $6x - 4$ |
| 4) $-9x$ | 5) $-\frac{1}{2}x - 1$ | 6) $\frac{2}{3}x - \frac{1}{2}$ |

5.2 Étudier le signe des polynômes suivants :

- | | | |
|---------------------|-----------------------|-----------------------|
| 1) $6x^2 - x - 2$ | 2) $8x^2 - 10x + 3$ | 3) $-x^2 + 6x - 9$ |
| 4) $-2x^2 + 7x + 4$ | 5) $25x^2 - 30x + 34$ | 6) $-3x^2 + 24x - 60$ |
| 7) $6x^2$ | 8) $6x^2 + 7x$ | 9) $8x^2 - 25$ |

5.3 Étudier le signe des polynômes suivants :

- | | |
|-----------------------------|--------------------------------|
| 1) $x(x - 1)$ | 2) $(x + 1)(x - 2)$ |
| 3) $(2 - x)(x + 3)(2x - 1)$ | 4) $x(2x - 3)(1 - 3x)$ |
| 5) $(x - 2)(6 - 2x)(4 - x)$ | 6) $(4 + x)(3x - 1)(2 - 2x)$ |
| 7) $(x + 2)(x^2 + 2x + 2)$ | 8) $x^2(x - 11)^{11}(6 - x)^3$ |

5.4 Étudier le signe des fractions rationnelles suivantes :

- | | | |
|-----------------------------------|------------------------------------|---|
| 1) $\frac{(x - 2)(x + 2)}{x - 1}$ | 2) $\frac{2x + 3}{(x - 1)(x + 2)}$ | 3) $\frac{(x - 2)(3 - x)}{(4 + x)(3x - 1)}$ |
|-----------------------------------|------------------------------------|---|

5.5 Étudier le signe des expressions suivantes :

- | | | |
|--------------------------|--------------------------|---|
| 1) $9x^2 - 4$ | 2) $8x^3 - 1$ | 3) $1 - x^4$ |
| 4) $4x^2 - 12x + 9$ | 5) $x^2 + 3x + 2$ | 6) $x^4 - 13x^2 + 36$ |
| 7) $\frac{1}{2 - x} + 1$ | 8) $x + \frac{2}{x - 3}$ | 9) $(x - 2)\left(\frac{3}{2 - x} - 3x\right)$ |

5.6 Résoudre les inéquations.

- | | |
|---|---|
| 1) $5(1 + 4x) > 7 + 12x$ | 2) $\frac{3 - x}{12} - \frac{x}{4} \geq 3 + \frac{5(x - 1)}{3}$ |
| 3) $2x - \frac{6x + 1}{2} - \frac{8x - 1}{3} + \frac{11x}{3} < 0$ | 4) $2x - \frac{2x}{9} \leq \frac{1}{9}\left(16x - \frac{3}{2}\right)$ |
| 5) $\frac{5x}{18} - \frac{4x - 3}{8} > \frac{9 - 2x}{9}$ | 6) $x - 7\left(\frac{x}{5} - \frac{x - 5}{4}\right) \geq -\frac{35}{4}$ |
| 7) $\frac{3x}{2} - \frac{2x}{3} \geq 5\left(\frac{x}{6} + 1\right) - 5$ | 8) $\frac{x - 2}{3} - \frac{5}{9}(x - 1) < \frac{x + 1}{4} - \frac{5}{2}$ |

5.7 Résoudre les inéquations.

- 1) $x^2 - 3x + 2 > 0$
- 2) $-x^2 + 3x + 4 > 0$
- 3) $x^2 - 10x + 40 > 0$
- 4) $4x^2 - 20x + 25 > 0$
- 5) $x^2 - 2x \leq 0$
- 6) $5x^2 > 0$
- 7) $5x^2 + 8x + 4 < 0$
- 8) $4x^2 > 16$
- 9) $(x - 2)^2 + 2x < -2$

5.8 Résoudre les inéquations.

- 1) $x^2 > x$
- 2) $x^3 < 4x$
- 3) $x^2 - 3x > -2$
- 4) $(x - 2)^2 > 4(x - 1)^2$
- 5) $x^3 - 1 < x^2 - 2x + 1$
- 6) $(x + 3)^2 > 5(x + 3)(x - 2)$
- 7) $(x - 1)^2(x - 2) < (x^2 - 1)(2 - x)$
- 8) $(2x - 1)^2 > 9(2 - x)^2$

5.9 Résoudre les inéquations.

- 1) $\frac{1}{x-1} + \frac{x}{x^2-1} > 0$
- 2) $x > \frac{1}{x}$
- 3) $\frac{4x-3}{x-1} \geq 2$
- 4) $\frac{x-1}{x^3} - \frac{x+1}{x^3-x^2} < \frac{3}{2x^2-x^3-x}$
- 5) $\frac{3x^2+12x+1}{2x^2+6x+1} \geq 2$
- 6) $\frac{2x}{4x^2-1} \geq \frac{2x+1}{4x^2-4x+1}$
- 7) $\frac{5x^2+2}{x^2-9} > \frac{5x-4}{x-3}$
- 8) $\frac{1}{x^2-2x} \leq 1 - \frac{2}{x}$
- 9) $\frac{x^2-3x+2}{x^2-7x+12} > 1$
- 10) $\frac{5}{x^2+5x+6} - \frac{2}{x^2-4} \geq \frac{3}{x^2-9}$

Réponses

- 5.1**
- 1) $\begin{array}{c} -\frac{1}{2} \\ | \\ \hline \end{array} \rightarrow$
 - 2) $\begin{array}{c} \frac{7}{3} \\ | \\ \hline \end{array} \rightarrow$
 - 3) $\begin{array}{c} \frac{2}{3} \\ | \\ \hline \end{array} \rightarrow$
 - 4) $\begin{array}{c} 0 \\ | \\ \hline \end{array} \rightarrow$
 - 5) $\begin{array}{c} -2 \\ | \\ \hline \end{array} \rightarrow$
 - 6) $\begin{array}{c} \frac{3}{4} \\ | \\ \hline \end{array} \rightarrow$

- 5.2**
- 1) $\begin{array}{c} -\frac{1}{2} \quad \frac{2}{3} \\ | \quad | \\ \hline \end{array} \rightarrow$
 - 2) $\begin{array}{c} \frac{1}{2} \quad \frac{3}{4} \\ | \quad | \\ \hline \end{array} \rightarrow$
 - 3) $\begin{array}{c} 3 \\ | \\ \hline \end{array} \rightarrow$
 - 4) $\begin{array}{c} -\frac{1}{2} \quad 4 \\ | \quad | \\ \hline \end{array} \rightarrow$
 - 5) $\begin{array}{c} + \\ | \\ \hline \end{array} \rightarrow$
 - 6) $\begin{array}{c} - \\ | \\ \hline \end{array} \rightarrow$
 - 7) $\begin{array}{c} 0 \\ | \\ \hline \end{array} \rightarrow$
 - 8) $\begin{array}{c} -\frac{7}{6} \quad 0 \\ | \quad | \\ \hline \end{array} \rightarrow$
 - 9) $\begin{array}{c} -\frac{5\sqrt{2}}{4} \quad \frac{5\sqrt{2}}{4} \\ | \quad | \\ \hline \end{array} \rightarrow$

- 5.3**
- 1) $\begin{array}{c} + & 0 & - & 1 & + \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$ 2) $\begin{array}{c} + & -1 & - & 2 & + \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$ 3) $\begin{array}{c} + & -3 & - & \frac{1}{2} & + & 2 & - \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 4) $\begin{array}{c} + & 0 & - & \frac{1}{3} & + & \frac{3}{2} & - \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$ 5) $\begin{array}{c} - & 2 & + & 3 & - & 4 & + \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$ 6) $\begin{array}{c} + & -4 & - & \frac{1}{3} & + & 1 & - \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 7) $\begin{array}{c} - & -2 & + \\ | & | & | \\ \hline \end{array} \rightarrow$ 8) $\begin{array}{c} - & 0 & - & 6 & + & 11 & - \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 5.4**
- 1) $\begin{array}{c} - & -2 & + & 1 & - & 2 & + \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$ 2) $\begin{array}{c} - & -2 & + & -\frac{3}{2} & - & 1 & + \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$ 3) $\begin{array}{c} - & -4 & + & \frac{1}{3} & - & 2 & + & 3 & - \\ | & | & | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 5.5**
- 1) $\begin{array}{c} + & -\frac{2}{3} & - & \frac{2}{3} & + \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$ 2) $\begin{array}{c} - & \frac{1}{2} & + \\ | & | & | \\ \hline \end{array} \rightarrow$ 3) $\begin{array}{c} - & -1 & + & 1 & - \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 4) $\begin{array}{c} + & \frac{3}{2} & + \\ | & | & | \\ \hline \end{array} \rightarrow$ 5) $\begin{array}{c} + & -2 & - & -1 & + \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$ 6) $\begin{array}{c} + & -3 & - & 2 & + & 2 & - & 3 & + \\ | & | & | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 7) $\begin{array}{c} + & 2 & - & 3 & + \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$ 8) $\begin{array}{c} - & 1 & + & 2 & - & 3 & + \\ | & | & | & | & | & | & | \\ \hline \end{array} \rightarrow$ 9) $\begin{array}{c} - & 1 & - & 2 & - \\ | & | & | & | & | \\ \hline \end{array} \rightarrow$
- 5.6**
- 1) $S =]\frac{1}{4}; +\infty[$ 2) $S =]-\infty; -\frac{13}{24}]$ 3) $S = \mathbb{R}$
 4) $S = \emptyset$ 5) $S = \emptyset$ 6) $S = [0; +\infty[$
 7) $S = \mathbb{R}$ 8) $S =]\frac{77}{17}; +\infty[$
- 5.7**
- 1) $S =]-\infty; 1[\cup]2; +\infty[$ 2) $S =]-1; 4[$
 3) $S = \mathbb{R}$ 4) $S = \mathbb{R} - \{\frac{5}{2}\} =]-\infty; \frac{5}{2}[\cup]\frac{5}{2}; +\infty[$
 5) $S = [0; 2]$ 6) $S = \mathbb{R} - \{0\} =]-\infty; 0[\cup]0; +\infty[$
 7) $S = \emptyset$ 8) $S =]-\infty; -2[\cup]2; +\infty[$
 9) $S = \emptyset$
- 5.8**
- 1) $S =]-\infty; 0[\cup]1; +\infty[$ 2) $S =]-\infty; -2[\cup]0; 2[$
 3) $S =]-\infty; 1[\cup]2; +\infty[$ 4) $S =]0; \frac{4}{3}[$
 5) $S =]-\infty; 1[$ 6) $S =]-3; \frac{13}{4}[$
 7) $S =]-\infty; 0[\cup]1; 2[$ 8) $S =]\frac{7}{5}; 5[$
- 5.9**
- 1) $S =]-1; -\frac{1}{2}[\cup]1; +\infty[$ 2) $S =]-1; 0[\cup]1; +\infty[$
 3) $S =]-\infty; \frac{1}{2}[\cup]1; +\infty[$ 4) $S =]0; \frac{1}{4}[$
 5) $S =]\frac{-3-\sqrt{7}}{2}; \frac{-3+\sqrt{7}}{2}[$ 6) $S =]-\frac{1}{2}; -\frac{1}{6}]$
 7) $S =]-\infty; -3[\cup]\frac{14}{11}; 3[$ 8) $S =]-\infty; 0[\cup]1; 2[\cup]3; +\infty[$
 9) $S =]\frac{5}{2}; 3[\cup]4; +\infty[$ 10) $S =]-\infty; -3[\cup]-2; 2[\cup]\frac{12}{5}; 3[$