2C - Suite - Croissance exponentielle - Log

§ 10.1 Introduction

ex. 10.1 à 10.8, 10.9 au tableau, 10.10, 10.11

ex 10.12,
plutôt non

§ 10.2 Eq. exponentielles

ex. 10.13 et 10.14

§ 10.3 Log. et éq. exp.

ex. 10.15 (les aider pour k) et l), 10.17 a), c), g), h), i), 10.18 (les aider pour le e)

10.19, ~~Revoir avec Coix pour introduire...~~

10.20 b) e) h), 10.21

§ 10.4 Log et éq. exp

ex : 10.22 (leur expliquer pourquoi le 4) est impossible), 10.23

§ 10.5

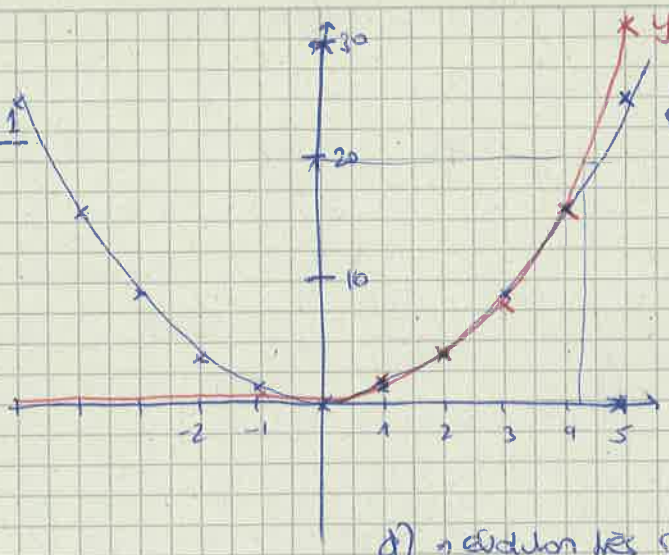
ex. 10.25 sauf d) et f), 10.26 a) b), c)

§ 10.6

ex 10.27, 10.29 b) (1/3) et c) (1/4), 10.30, 10.32, 10.33.

10.1

a)



b) $y = 2^x$

c) 1) $x = \pm 4$
2) $x = 4$
3) $x \approx \pm 4,5$
4) $x \approx 4,2$
5) $x = \pm 1$
6) $x = 0$

d) Solution les rapide.

Modèle 1

Après 1 an : $10\,000 + 10\,000 \cdot \frac{6}{100} = 10\,600$ (+600)

$10\,000\,(1 + 0,06)$

Après 2 ans : $10\,600\,(1 + 0,06) = 11\,236$ (+636)

Après 3 ans : $11\,236\,(1 + 0,06) = 11\,910,16$ (+674,16)

Modèle 2

a) Comme dans le modèle 1, on va multiplier chaque fois par 1,0225

⇒ Après n années : $5000 \cdot 1,0225^n = C(n)$

b) Si $n = 15$: $C(15) = 5000 \cdot 1,0225^{15} \approx 6981,-$

10.2

a) $7500 \cdot (1,0375)^n = C(n)$

b) $C(5) \approx 7500 \cdot (1,0375)^5 \approx 9015,75$

10.3

$C(n) = C(1,0275)^{13}$

On cherche C tq. $C(1,0275)^{13} = 10671,5$

$\Rightarrow C = \frac{10\,671,5}{1,0275^{13}} \approx 7500,-$

10.4

$C(1,0325)^{15} = 20'000 \Rightarrow C = \frac{20000}{1,0325^{15}} \approx 12378,82$

10.5

$8000(1+i)^{12} = 12'000 \Rightarrow (1+i)^{12} = \frac{12000}{8000} = 3/2$

$\Rightarrow 1+i = \sqrt[12]{3/2} \Rightarrow i = \sqrt[12]{1,5} - 1 \approx 0,0344 \approx 3,44\%$

10.6

a) $C_0 = \frac{C(n)}{(1+t)^n}$ b) $(1+t)^n = \frac{C(n)}{C_0} \Rightarrow t = \sqrt[n]{\frac{C(n)}{C_0}} - 1$

Modèle 3

a) $V(n) = V_0\left(1 - \frac{15}{100}\right)^n = V_0 \cdot 0,85^n$

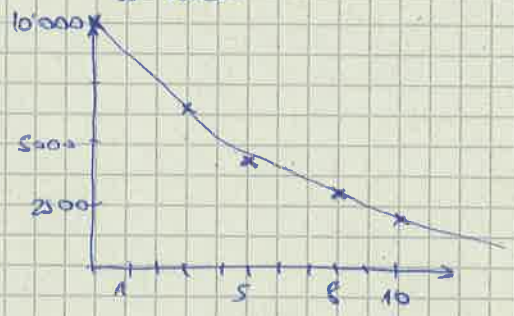
b) $V(2) = V_0 \cdot 0,85^2 \approx 0,72 \cdot V_0 = \underbrace{(1-0,38) \cdot V_0}_{\text{il a perdu 38\% de sa valeur}}$

10.7

a) $V(n) = V_0 \cdot 0,85^n$

b) $V(8) = 10'000 \cdot 0,85^8 \approx 2724,90$

$V(10) = 10'000 \cdot 0,85^{10} \approx 1968,70$



10.8

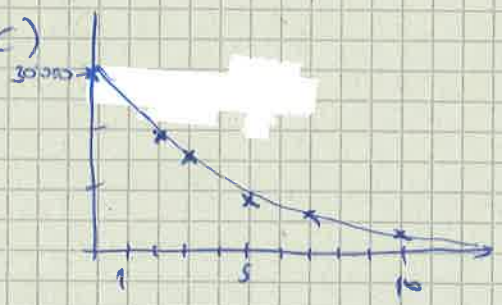
a) $V(n) = 300'000 \cdot 0,8^n$

b) $V(2) = 300'000 \cdot 0,8^2 \approx 192'000$

$V(3) = 300'000 \cdot 0,8^3 \approx 153'600$

$V(5) = 300'000 \cdot 0,85^5 = 98'304$

c)



10.9

a) $V(0) = V_0 = 35'000$ et $\underbrace{V(5)}_{=56'800} = 35'000 \cdot x^5 \Rightarrow x = \sqrt[5]{\frac{56800}{35000}} \approx 1,1017$

$\Rightarrow$ intérêt de 10,17%

et $V(n) = 35000 \cdot 1,1017^n$

b) $V(\underbrace{2005-1977}_{28}) = \underbrace{35'000 \cdot 1,1017^{28}}_{527037,90} \approx 526'790,20$ ← sans les arrondis

↑ avec arrondis

10.10

$250\,000 \cdot (1-t)^5 = 110\,000$

$(1-t)^5 = 11/25$ d'où $t = -\sqrt[5]{11/25} + 1 \approx 15.14\,\%$

10.11

a) $Q(n) = Q_0 \cdot \left(\frac{49}{50}\right)^n$

b) $Q(5) = 100 \left(\frac{49}{50}\right)^5 \approx 90,39$ u

$Q(10) = 100 \left(\frac{49}{50}\right)^{10} \approx 81,71$ u

10.12

a) $Q(n) = Q_0 \cdot \left(\frac{8}{10}\right)^n = Q_0 \left(\frac{4}{5}\right)^n = Q_0 \cdot 0,8^n$

b)

Module 4

a) $2^{2x} = \underbrace{64}_{=2^6}$ donc $2x = 6 \Rightarrow \underline{x = 3}$

b) $10^x = \underbrace{0,01}_{=\frac{1}{100}=\frac{1}{10^2}=10^{-2}} \Rightarrow \underline{x = -2}$

c) $10^{3x+2} = 10^{-1} \Rightarrow 3x+2 = -1 \Rightarrow 3x = -3 \Rightarrow \underline{x = -1}$

d) $4 \cdot 3^{x^2-1} = 108 \Leftrightarrow 3^{x^2-1} = \underbrace{27}_{=3^3} \Leftrightarrow x^2 - 1 = 3$

$\Rightarrow x^2 = 4 \Rightarrow \underline{x = \pm 2}$

e) $\frac{\underbrace{4}_{2^2}^{3x+1}}{2^{6x+2}} = \underbrace{8}_{2^3} \Rightarrow 6x+2 = 3 \Rightarrow 6x = 1 \Rightarrow \underline{x = \frac{1}{6}}$

10.13

a) $\underline{x = 2}$ b) $\underline{x = -2}$ c) $3^{x-1} = 9 \quad x-1 = 2, \ \underline{x = 3}$

d) $2^{2x} = 2^8$ $\underline{x = 4}$ e) $2^{2x} = 2^3$ $\underline{x = 3/2}$ f) $-x = 3$ $\underline{x = -3}$

g) $3^x = 3^{4x+6} \Rightarrow 3x = -6 \Rightarrow \underline{x = -2}$

h) $10^{x+2} = 10^{2x} \Rightarrow \underline{x = 2}$

i) $3^{3x+2} = 3^{2x} \Rightarrow \underline{x = -2}$

j) $2x + 1 = 0 \quad \underline{x = -1/2}$

k) $x + 7 = 10x + 4 \rightarrow 9x = 3 \Rightarrow \underline{x = 1/3}$

l) $8x^2 + 4 = 4 - 12x + 9x^2 \rightarrow x^2 - 12x = 0$

$\underline{S = \{0; 12\}}$

Module 5

$2^x = 20$

On a $2^4 = 16$ et $2^5 = 32$ donc x entre 4 et 5.

$2^{4.5} = 22.6$

$2^{4.3} = 19.7$ donc $4.3 < x < 4.5$

$2^{4.4} = 21.11 \Rightarrow 4.3 < x < 4.4$

$2^{4.35} = 20.4 \Rightarrow 4.3 < x < 4.35$

$2^{4.33} = 20.11$

$2^{4.32} = 19.97 \rightarrow 4.32 < x < 4.33$ etc...

10.14

a) $10^{2.5} \approx 316 \qquad 10^{2.4} \approx 251$

$10^{2.48} \approx 301.95 \qquad 10^{2.477} \approx 299.9$ ...

b) $\text{Log } 300 \approx 2.477...$

Module 6

a) c'est 3 car $10^3 = 1000$

b) $\log_{10}(0,01) = x \Leftrightarrow 10^x = \underbrace{0,01}_{10^{-2}} \Leftrightarrow \underline{x = -2}$

10.15

a) $\log_{10}(10^6) = 6$

b) $\log_{10}(10^0) = 0$

c) $\log_{10}(10^{-5}) = -5$

d) $\log_{10}(10^{-3}) = -3$

e) $10^3 = x \Rightarrow x = 1000$

f) $10^{-1} = x \Rightarrow x = 0,1$

g) $10^{-4} = x \Rightarrow x = 0,0001$

h) $\log_{10}(10^{1/2}) = \frac{1}{2}$

i) $\log_{10}(10^{-2}) = -2$

j) $10^0 = x \Rightarrow x = 1$

k) $\log_{10}(\log_{10}(x)) = \underbrace{10^0}_{=1}$

donc $\log_{10}(x) = 10$

donc $x = 10^{10}$

l) $5 - x = 10^8 \Rightarrow x = 5 - 10^8$

Module 7

On cherche x tel que $10^x = 45$ donc

$x = \log_{10}(45) \overset{\text{machine}}{\simeq} 1,653$ donc $45 \simeq 10^{1,653}$

10.17

a) $\log(3) \simeq 0,477$ $\quad 3 \simeq 10^{0,477}$

b) $54,5 \simeq 10^{1,736}$

c) $0,22 \simeq 10^{-0,658}$

d) $1,2 \simeq 10^{0,0792}$

e) $3,7 \simeq 10^{0,568}$

f) $0,37 \simeq 10^{-0,432}$

g) $8,32 \simeq 10^{0,920}$

h) $83,2 \simeq 10^{1,920}$

i) $832 \simeq 10^{2,920}$

Tableau :

x	100	$10^3 = 1000$	17	0,01	$\sqrt{10} \simeq 3,16$
$\log$	2	3	$\simeq 1,23$	-2	0,5

10.18

a) $x = \log(8) \simeq 0,903$

b) $x = \log(0,65) \simeq -0,187$

c) $x = 10^{1,5} \simeq 31,623$

d) $x = 10^{-0,27} \approx 0,537$

e) $2x = \log(0,7) \Rightarrow x = \frac{\log(0,7)}{2} \approx -0,077$

f) $\log_{10}(x) = 2,5 \Rightarrow x = 10^{2,5} \approx 316,228$

g) On a soit $10^x = 0$ (impossible), soit $10^x = 4$ donc $x = \log(4) \approx 0,602$

$S = \{0,602\}$

h) $3x = \log(25) \Rightarrow x = \frac{\log(25)}{3} \approx 0,466$

i) On a soit $10^{-x} = 0$ ⇒ impossible, soit $10^{-x} = 8$ donc

$-x = \log(8) \Rightarrow x = -\log(8) \approx -0,903 \Rightarrow S = \{-0,903\}$

10.19

$\left(1 + \frac{1}{10}\right)^{10} \approx 2,594$, $\left(1 + \frac{1}{100}\right)^{100} \approx 2,705$

$\left(1 + \frac{1}{1000}\right)^{1000} \approx 2,717$; $\left(1 + \frac{1}{10000}\right)^{10000} \approx 2,718$

Modèle 8

$7,39 = 10^x \Leftrightarrow x = \log(7,39) \approx 0869 \Rightarrow 7,39 \approx 10^{0,869}$

$0,01 = 10^{-2}$

et $7,39 = e^x \Leftrightarrow x = \ln(7,39) \approx 2 \Rightarrow 7,39 \approx e^2$

$0,01 = e^x \Leftrightarrow x = \ln(0,01) \approx -4,605 \Rightarrow 0,01 \approx e^{-4,605}$

10.20

a) $3 = e^{\ln(3)} \approx e^{1,0986}$

$a = e^{\ln(a)}$ etc..

10.21

a) $x = \ln(7) \approx 1,946$ b) $x = \ln(0,65) \approx -0,431$

c) $x \approx 1,0986$ d) $x = e^3 \approx 20,086$

e) $-0,01x = \ln(0,7)$

$x \approx \frac{\ln(0,7)}{-0,01} \approx 35,667$

Thade 9:

a) $\log(3^x) = \log(24)$

$= x \cdot \log(3) \rightarrow x = \frac{\log(24)}{\log(3)} \approx 2,893$

b) $\log(5^{0,3x+1}) = \log(18)$

$(0,3x+1) \cdot \log(5) = \log(18)$

$0,03x = \frac{\log(18)}{\log 5} - 1 \rightarrow x = \frac{1}{0,3} \cdot \left(\frac{\log(18)}{\log(5)} - 1\right) \approx 2,653$ ~~5,986~~

10.22

a) $x = \frac{2}{\log(2)} \approx 6,644$

b) $x = \log(5) \approx 0,699$

c) $2^{4x} = 2^3 \rightarrow x = 3/4 = 0,75$

d) $x = 1 + \frac{1}{\log(3)} \approx 3,096$

e) $2x + 3 = \log(7)/\log(5) \rightarrow x = \frac{1}{2}\left(\frac{\log(7)}{\log(5)} - 3\right) \approx -0,895$

f) $x = \frac{2}{\log(1,04)} \approx$ ~~51,287~~ 117,417

g) $x = 4 + \frac{\log(20)}{\log(4)} \approx 6,161$

h) $\log(-25)$ Ikke! $5^{8x} > 0$ og $-25 < 0$.

i) $x = \frac{\log(149)}{\log(12)} \approx 2,014$

Thade 10

$(3x+1)\log(2) = (2x-2)\log(3)$

$(3\log 2 - 2\log(3))x = -\log(2) - 2\log(3)$

$x = \frac{-\log(2) - 2\log(3)}{3\log(2) - 2\log(3)} \approx -0,676$

10.23

a) $(x-1)\log(2) = x\log(3)$

$x(\log(2) - \log(3)) = \log(2)$

$\Leftrightarrow x = \dfrac{\log(2)}{\log(2) - \log(3)} \approx -1,7095$

b) $(x+1)\log(4) = 3x\log(6)$

$\Leftrightarrow (\log 4 - 3\log 6)\cdot x = -\log(4)$

$x = \dfrac{-\log(4)}{\log(4) - 3\log(6)} \approx 0,348$

c) $(x+4)\cdot\log(2) = (x-1)\cdot\log(3)$

$(\log(2) - \log(3))\cdot x = -4\log(2) - \log(3)$

$x \approx \dfrac{-4\log(2) - \log(3)}{\log(2) - \log(3)} \approx 9,548$

10.24

$\text{Log}(a^x) \overset{\text{déf.}}{=} \text{Log}\left(\left(10^{\log(a)}\right)^x\right) = \text{Log}\left(10^{\log(a)\cdot x}\right) = \log(a)\cdot x$

$= x\log(a)$.

$a = 10^y \Leftrightarrow y = \text{Log}(a)$

donc $a = 10^{\text{Log}(a)}$

Modèle 11

a) $\frac{x}{5} = 3,2^{4y} \Leftrightarrow \text{Log}\left(\frac{x}{5}\right) = 4y\,\text{Log}(3,2) \Leftrightarrow$

$y = \dfrac{\text{Log}(x/5)}{4\text{Log}(3,2)}$

b) $\frac{x}{2} = \ln(3y) \Leftrightarrow 3y = e^{x/2} \Leftrightarrow y = \frac{1}{3}e^{x/2}$

10.25

a) $y = \text{Log}(10)$

b) $\text{Log}(x) = (y+1)\text{Log}(3) \Rightarrow y = \dfrac{\text{Log}(x)}{\text{Log}(3)} - 1$

c) $\ln(x) = 10y - 2 \Leftrightarrow y = \frac{1}{10}(\ln(x) + 2)$

d) $\text{Log}(x-1) = 4,8y\,\text{Log}(95) \Rightarrow y = \dfrac{1}{0,8}\dfrac{\text{Log}(x-1)}{\text{Log}(95)}$

10.25 (suite)

e) $\text{Log}(4/3) = n\,\text{Log}(1{,}02) \Rightarrow n = \dfrac{\text{Log}(4/3)}{\text{Log}(1{,}02)}$

f) $n = \dfrac{\text{Log}(C(n)/C_0)}{\text{Log}(1+t)}$

10.26

a) $y = 10^x$

b) $10^{5x} = 2y \Rightarrow y = \frac{1}{2}\,10^{5x}$

c) $y = \frac{1}{4}\,e^{x/3}$

d) $y = \dfrac{1}{0{,}2}\,e^{\frac{x+3{,}5}{0{,}25}}$ et $\dfrac{1}{0{,}2} = \dfrac{1}{2/10} = 5$

et $\dfrac{x+7/2}{1/4} = \dfrac{4x+14}{1} = 4x+14$

$\Rightarrow y = 5e^{4x+14}$

Modèle 12

a) $P(3) = 1 - e^{-0{,}21\cdot 3} \simeq 0{,}467 = 46{,}7\,\%$

b) On cherche t tq $1 - e^{-0{,}21\cdot t} = 0{,}9$

$\Leftrightarrow 0{,}1 = e^{-0{,}21\cdot t}$

$\Leftrightarrow t = \dfrac{1}{-0{,}21}\cdot \ln(0{,}1) \simeq 10{,}96 \Rightarrow$ 11 jours.

10.27

On cherche t tq $20000\,e^{0{,}05\,t} = 40000$

$\Rightarrow t = \dfrac{1}{0{,}05}\cdot \ln(2) \simeq 13{,}86$ années

$= 13$ ans et ~ 10 mois

10.28

a) M1 : 4 milliards dans les 3 cas.

b) 1990 : $\left.\begin{matrix} M1 \\ M2 \\ M3 \end{matrix}\right\}$ cf. corrigé.

c) M1 : $4\cdot e^{0{,}02t} = 10$ ou $t = \dfrac{1}{0{,}02}\cdot \ln\left(\dfrac{10}{4}\right) \simeq 45{,}8$

$1976 + 46 = 2022$

M2 : $20 - 16\,e^{-0,005\,t} = 10$

$10/16 : e^{-0,005t} \Rightarrow t = \frac{1}{-0,005} \cdot \ln\left(\frac{10}{16}\right) \approx 94$

$1976 + 94 = \underline{2070}$.

M3 $20 = 10\left(1 + 4\,e^{-0,025t}\right)$

$\ln(1/4) = -0,025t \Rightarrow t = \frac{1}{-0,025} \cdot \ln(1/4) \approx 55,45$

$1976 + 55 = \underline{2031}$.

Modèle 13

$V(n) = 405\,000\ 0,8^n$

On veut n tq $0,8^n = \frac{1}{2} \Rightarrow n = \frac{\log(1/2)}{\log(0,8)} \approx 3,1$

$\Rightarrow \sim 3$ ans

10.29

* On veut n tq $0,8^n = \frac{1}{2} \Rightarrow n \approx 3,1 \sim \underline{3 \text{ ans}}$
* $0,8^n = 1/3 \Rightarrow n = \log(1/3)/\log(0,8) \approx 4,92 \sim \underline{5 \text{ ans}}$
* $0,8^n = 1/4 \Rightarrow n \approx 6,21 \sim \underline{6 \text{ ans}}$
* $0,8 = 1/5 \Rightarrow n \approx 7,21 \sim \underline{7 \text{ ans}}$

10.30

$P(n) = P_0 \cdot 1,02^n$

a) $P(n) = 10\,000 \cdot 1,02^n$ on veut n tq.

$10\,000 \cdot 1,02^n = 20\,000$

$\Leftrightarrow n = \log(2)/\log(1,02) \approx \underline{35 \text{ ans}}$

b) Idem

Modèle 14

On veut t tq. $1,085^t = 2 \Leftrightarrow n = \frac{\log(2)}{\log(1,085)} \approx 8,5$

$\Rightarrow$ dans et demi.

10.31

a) $Q_0 \cdot 0,98^n = \frac{3}{4} Q_0 \Rightarrow$ i) $n = \frac{\log(0,75)}{\log(0,98)} \approx \underline{14,24}$

ii) $n \approx \frac{\log(0,5)}{\log(0,98)} \approx \underline{34,3}$

iii) $n \approx \frac{\log(0,25)}{\log(0,98)} \approx \underline{68,61}$

10.31 (suite)

b) vu a) ~ 34 ans.

10.32

$P(n) = P_0 \cdot 0{,}95^n$

$P_0 \cdot 0{,}95^n = \frac{1}{2} P_0 \Rightarrow n = \frac{\log(1/2)}{\log(0{,}95)} \approx 13{,}5$ ans

10.33

a) $P(n) = \frac{P_0}{200\,000} \cdot 1{,}0325^n$

On veut P_0 tq $200\,000 = P_0 \cdot 1{,}0325^{15}$

$\Rightarrow P_0 = \frac{200\,000}{1{,}0325^{15}} \approx 123\,788$ m³.