Quelques réponses : Thème 9

Exercice 9.1:

a) $\left(2^2\right)^3 = 2^{2\cdot3} = 2^6 = \mathbf{64}$ **b)** $2^{\left(2^3\right)} = 2^8 = \mathbf{256}$

c) $\left(2^3\right)^2 = 2^{3\cdot2} = 2^6 = \mathbf{64}$ **d)** $2^3 - 3^2 = 8 - 9 = \mathbf{-1}$

e) $3^2 + 3^4 = 9 + 81 = \mathbf{90}$ **f)** $10^3 + 10^2 = 1000 + 100 = \mathbf{1100}$

g) $\left(\frac{1}{3}\right)^4 = \frac{1^4}{3^4} = \mathbf{\frac{1}{81}}$ **h)** $\left(-\frac{2}{5}\right)^3 = \left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{5^3} = \mathbf{\frac{-8}{125}}$

i) $\left(-\frac{3}{2}\right)^4 = \left(\frac{-3}{2}\right)^4 = \frac{(-3)^4}{2^4} = \mathbf{\frac{81}{16}}$ **j)** $\left(\sqrt{2}\right)^4 = \sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2} = 2\cdot2 = \mathbf{4}$

k) $\left(\sqrt{5}\right)^6 = \ldots = 5^3 = \mathbf{125}$ **l)** $\left(-\frac{1}{\sqrt{3}}\right)^8 = \left(\frac{-1}{\sqrt{3}}\right)^8 = \ldots = \frac{1}{3^4} = \mathbf{\frac{1}{81}}$

Exercice 9.2:

a) $\frac{2^6}{2^2} = 2^{6-2} = 2^4 = \mathbf{16}$ **b)** $\frac{-2^6}{2^3} = -\frac{2^6}{2^3} = -2^{6-3} = -2^3 = \mathbf{-8}$

c) $\frac{(-2)^6}{2^3} = \frac{2^6}{2^3} = 2^{6-3} = 2^3 = \mathbf{8}$ **d)** $\frac{2^{10}}{2^{12}} = \frac{1}{2^{12-10}} = \frac{1}{2^2} = \mathbf{\frac{1}{4}}$

e) $\frac{2^4}{-2^5} = -\frac{2^4}{2^5} = -\frac{1}{2^{5-4}} = \mathbf{-\frac{1}{2}}$ **f)** $\frac{2^4}{(-2)^5} = -\frac{2^4}{2^5} = -\frac{1}{2^{5-4}} = \mathbf{-\frac{1}{2}}$

g) $\left(\frac{2}{2^3}\right)^2 = \left(\frac{1}{2^2}\right)^2 = \frac{1^2}{2^4} = \mathbf{\frac{1}{16}}$ **h)** $\left(-\frac{2^7}{2^6}\right)^3 = \left(-2^{7-6}\right)^3 = (-2)^3 = \mathbf{-8}$

i) $\left(\frac{2^0}{2^2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1^2}{4^2} = \mathbf{\frac{1}{16}}$ **j)** $\frac{2^{16}}{4^7} = \frac{2^{16}}{\left(2^2\right)^7} = \frac{2^{16}}{2^{14}} = 2^{16-14} = 2^2 = \mathbf{4}$

k) $\frac{9^5}{3^{10}} = \frac{\left(3^2\right)^5}{3^{10}} = \frac{3^{10}}{3^{10}} = \mathbf{1}$ **j)** $\left(\frac{5^3}{25^2}\right)^2 = \left(\frac{5^3}{5^4}\right)^2 = \left(\frac{1}{5}\right)^2 = \mathbf{\frac{1}{25}}$

Exercice 9.3:

a) $2^{-1}\cdot 2^{-2} = 2^{-1-2} = 2^{-3} = \mathbf{\frac{1}{8}}$ **b)** $3^{-4}\cdot 3^4 = 3^{-4+4} = 3^0 = \mathbf{1}$

c) $4^0\cdot 4^{-5}\cdot 4^3 = 4^{-2} = \mathbf{\frac{1}{16}}$ **d)** $a^{-3}\cdot a^4 = a^{-3+4} = a^1 = \boldsymbol{a}$

e) $\frac{2^{-3}}{3^2} = \frac{\frac{1}{2^3}}{3^2} = \frac{1}{2^3}\cdot\frac{1}{3^2} = \frac{1}{8}\cdot\frac{1}{9} = \mathbf{\frac{1}{72}}$ **f)** $\left(\frac{1}{2}\right)^{-2} = \frac{1}{(1/2)^2} = \frac{1}{1/4} = 1\cdot 4 = \mathbf{4}$

g) $\frac{2^3}{3^{-2}} = \frac{8}{\frac{1}{3^2}} = 8\cdot 3^2 = \mathbf{72}$ **h)** $\left(-\frac{2}{5}\right)^{-2} = \left(\frac{-2}{5}\right)^{-2} = \left(\frac{5}{-2}\right)^2 = \frac{5^2}{(-2)^2} = \mathbf{\frac{25}{4}}$

i) $\left(\frac{1}{5}\right)^2\cdot\left(\frac{2}{5}\right)^{-2} = \left(\frac{1}{5}\right)^2\cdot\left(\frac{5}{2}\right)^2 = \frac{1}{2^2} = \mathbf{\frac{1}{4}}$

QUELQUES RÉPONSES AUX EXERCICES

Exercice 9.4:

a) $2^3\cdot 2^4\cdot 2^{-5} = 2^{3+4-5} = 2^{\mathbf{2}}$ **b)** $\left(3^4\right)^2 = 3^8 = \frac{1}{3^{\mathbf{-8}}}$

c) $\frac{3^8}{3^2} = 3^6 = \left(3^2\right)^3 = 9^{\mathbf{3}}$ **d)** $\frac{3^2}{3^6} = 3^{-4} = \left(3^2\right)^{-2} = 9^{\mathbf{-2}}$

e) $\frac{5}{25^2} = \frac{5}{5^4} = 5^{\mathbf{-3}}$ **f)** $2^3\cdot 2^9 = 2^{12} = \left(2^2\right)^6 = 4^{\mathbf{6}}$

g) $36^2\cdot 6^{-2} = 6^4\cdot 6^{-2} = 6^2 = \frac{1}{6^{\mathbf{-2}}}$ **h)** $\left(-\frac{1}{5}\right)^{-3} = \left(\frac{-1}{5}\right)^{-3} = \left(\frac{5}{-1}\right)^3 = (\mathbf{-5})^3$ ou $\mathbf{-5^3}$

i) $\left(\frac{1}{25}\right)^{-3} = \left(\frac{25}{1}\right)^3 = \left(5^2\right)^3 = 5^{\mathbf{6}}$ **j)** $\left(\frac{2^3}{2^5}\right)^{-3} = \left(\frac{2^5}{2^3}\right)^3 = \left(2^2\right)^3 = 2^{\mathbf{6}}$

k) $\left(\frac{2^{-1}}{2^3}\right)^2 = \left(\frac{1}{2^4}\right)^2 = \left(2^{-4}\right)^2 = 2^{\mathbf{-8}}$ **l)** $\left(\frac{3^{-4}}{9^2}\right)^2 = \frac{3^{-8}}{9^4} = \frac{\left(3^2\right)^{-4}}{9^4} = \frac{9^{-4}}{9^4} = 9^{\mathbf{-8}}$

Exercice 9.5: Dans les 2 cas, il suffit de montrer que les valeurs des 2 côtés du "=" sont bien identiques.

Exercice 9.6:

a) $\sqrt{0} = 0$ car $0^2 = 0$ **b)** $\sqrt{0{,}04} = 0{,}2$ car $0{,}2^2 = 0{,}04$

c) $\sqrt{0{,}0009} = 0{,}03$ car $0{,}03^2 = \ldots$ **d)** $\sqrt[3]{1000} = 10$ car $10^3 = 1000$

e) $\sqrt[5]{32} = 2$ car $2^5 = 32$ **f)** $\sqrt[4]{16} = 2$ car $2^4 = 16$

g) $\sqrt[3]{0{,}027} = 0{,}3$ car $0{,}3^3 = \ldots$ **h)** $\sqrt[4]{0{,}0001} = 0{,}1$ car $0{,}1^4 = 0{,}0001$

i) $\sqrt[3]{0{,}125} = 0{,}5$ car $0{,}5^3 = \ldots$ **j)** $\sqrt{0{,}0001} = 0{,}01$ car $0{,}01^2 = 0{,}0001$

k) $\sqrt[3]{0{,}000008} = 0{,}02$ car $0{,}02^3 = 0{,}000008$

Exercice 9.7:

a) $\sqrt[3]{-27} = -3$ car $(-3)^3 = -27$ **b)** $\sqrt[5]{-1} = -1$ car $(-1)^5 = -1$

c) $\sqrt[2]{-4}$ est non définie **d)** $\sqrt[3]{(-2)^3} = -2$

e) $\sqrt{(-2)^2} = 2$ car $\sqrt{(-2)^2} = \sqrt{4}$ **f)** $\sqrt[3]{-0{,}125} = -0{,}5$ car $(-0{,}5)^3 = -0{,}125$

g) $\sqrt[3]{-0{,}027} = -0{,}3$ car $\ldots$ **h)** $\left(\sqrt[4]{-1}\right)^2$ est non définie

i) $\sqrt{(-2)^4} = 4$

Exercice 9.8:

a) $\sqrt[3]{\frac{1}{4}}\cdot\sqrt[3]{\frac{1}{2}}=\sqrt[3]{\frac{1}{8}}=\mathbf{\frac{1}{2}}$ **b)** $\sqrt{2^2}=\mathbf{2}$

c) $\sqrt{9+1}=\sqrt{\mathbf{10}}$ **d)** $\sqrt[4]{8}\cdot\sqrt[4]{32}=\sqrt[4]{256}=\mathbf{4}$

e) $\sqrt{2^6}=\sqrt{64}=\mathbf{8}$ **f)** $\sqrt{9}+\sqrt{3}=\mathbf{3+\sqrt{3}}$

g) $\sqrt[3]{-\frac{27}{1000}}=\frac{\sqrt[3]{-27}}{\sqrt[3]{1000}}=\mathbf{\frac{-3}{10}}$ **h)** $\left(\sqrt[4]{0{,}0001}\right)^4=\mathbf{0{,}0001}$

i) $\sqrt[5]{2^{10}}=\sqrt[5]{\left(2^2\right)^5}=2^2=\mathbf{4}$

Exercice 9.9:

a) $\sqrt{2700}=\sqrt{100}\cdot\sqrt{27}\cong 10\cdot 5{,}19\cong 51{,}9$

b) $\sqrt{27'000}=\sqrt{100}\cdot\sqrt{270}\cong 10\cdot 16{,}43\cong 164{,}3$

c) $\sqrt{270'000}=\sqrt{10'000}\cdot\sqrt{27}\cong 100\cdot 5{,}19=519$

d) $\sqrt{0{,}27}=\sqrt{\frac{27}{100}}=\frac{\sqrt{27}}{10}\cong 0{,}519$

e) $\sqrt{0{,}027}=\sqrt{\frac{270}{10'000}}=\frac{\sqrt{270}}{100}=0{,}1643$

f) $\sqrt{0{,}000027}=\sqrt{\frac{27}{1'000'000}}=\frac{\sqrt{27}}{1000}\cong\frac{5{,}19}{1000}\cong 0{,}00519$

Exercice 9.10:

a) $\sqrt[3]{27'000}=\sqrt[3]{1'000}\cdot\sqrt[3]{27}=10\cdot 3=30$

b) $\sqrt[3]{270'000}=\sqrt[3]{1'000}\cdot\sqrt[3]{270}\cong 10\cdot 6{,}46=64{,}6$

c) $\sqrt[3]{2'700'000}=\sqrt[3]{1'000}\cdot\sqrt[3]{2'700}\cong 10\cdot 13{,}92=139{,}2$

d) $\sqrt[3]{0{,}27}=\sqrt[3]{\frac{270}{1'000}}=\frac{\sqrt[3]{270}}{10}\cong\frac{6{,}46}{10}\cong 0{,}646$

e) $\sqrt[3]{0{,}027}=\sqrt[3]{\frac{27}{1'000}}=\frac{\sqrt[3]{27}}{10}=\frac{3}{10}=0{,}3$

f) $\sqrt[3]{0{,}00027}=\sqrt[3]{\frac{270}{1'000'000}}=\frac{\sqrt[3]{270}}{100}\cong\frac{6{,}46}{100}=0{,}0646$

Exercice 9.11:

a) $\sqrt{300}\cong 17{,}3$

b) $\frac{3}{\sqrt{3}}\cong 1{,}73$

c) $\sqrt{\frac{4}{3}}=\frac{2}{\sqrt{3}}=\frac{2\cdot\sqrt{3}}{3}\cong\frac{3{,}46}{3}\cong 1{,}15$

Exercice 9.12:

a) $10\sqrt{3}$ **b)** $\sqrt{3}$ **c)** $\frac{2\sqrt{3}}{3}$ **d)** $2^3\sqrt{2}=8\sqrt{2}$

e) $20\sqrt{10}$ **f)** $\frac{\sqrt{26}}{2}$ **g)** $5\sqrt{3}$ **h)** $2+\sqrt{2}$

i) $\frac{3\sqrt{3}}{2}$ **j)** $\frac{2\sqrt{3}+3}{3}$ **k)** $9\sqrt{2}$ **l)** $\frac{\sqrt{2}}{2}$

Exercice 9.13: **a)** 5 **b)** 3 **c)** 9

Exercice 9.14:

a) $5^{1/2}=\sqrt{\mathbf{5}}$ **b)** $4^{3/2}=\left(4^{1/2}\right)^3=\left(\sqrt{4}\right)^3=\mathbf{8}$

c) $16^{1/4}=\left(2^4\right)^{1/4}=\mathbf{2}$ **d)** $32^{1/10}=\left(32^{1/5}\right)^{1/2}=2^{1/2}=\sqrt{\mathbf{2}}$

e) $9^{3/2}=\left(9^{1/2}\right)^3=3^3=\mathbf{27}$ **f)** $25^{0{,}5}=25^{1/2}=\mathbf{5}$

g) $\left(\frac{1}{9}\right)^{-\frac{1}{2}}=9^{1/2}=\mathbf{3}$ **h)** $\left(\frac{27}{8}\right)^{\frac{2}{3}}=\left(\sqrt[3]{\frac{27}{8}}\right)^2=\left(\frac{3}{2}\right)^2=\mathbf{\frac{9}{4}}$

i) $2^{-1/2}=\frac{1}{2^{1/2}}=\frac{1}{\sqrt{2}}=\mathbf{\frac{\sqrt{2}}{2}}$ **j)** $9^{-1{,}5}=\frac{1}{9^{3/2}}=\frac{1}{\left(9^{1/2}\right)^3}=\mathbf{\frac{1}{27}}$

k) $5^{6/5}=\left(5^6\right)^{1/5}=\sqrt[5]{5^6}=\mathbf{5\sqrt[5]{5}}$ **l)** $\frac{2}{8^{1/2}}=\frac{2}{\sqrt{8}}=\frac{2}{2\sqrt{2}}=\mathbf{\frac{\sqrt{2}}{2}}$

Exercice 9.15:

a) $3^{1/5}$ **b)** $5^{6/11}$ **c)** $3^{3/9}=3^{1/3}$

d) $\mathbf{2}^{-1/2}$ **e)** $\left(\frac{3}{2}\right)^{1/2}$ **f)** $3^{9/3}=3^3$

Exercice 9.16:

a) $\sqrt[4]{2}\cdot\sqrt{2}=2^{1/4}\cdot 2^{1/2}=2^{1/4+1/2}=2^{3/4}=\sqrt[4]{2^3}=\sqrt[4]{\mathbf{8}}$

b) $\sqrt[4]{3}\cdot\sqrt[3]{3^2}=3^{1/4}\cdot 3^{2/3}=3^{1/4+2/3}=3^{11/12}=\sqrt[12]{\mathbf{3^{11}}}$

c) $\sqrt[3]{25}\cdot\sqrt[6]{5}=\sqrt[3]{5^2}\cdot\sqrt[6]{5}=5^{2/3}\cdot 5^{1/6}=5^{2/3+1/6}=5^{5/6}=\sqrt[6]{\mathbf{5}^5}$

d) $\frac{\sqrt{3}}{\sqrt[3]{3}}=\frac{3^{1/2}}{3^{1/3}}=3^{1/2-1/3}=3^{1/6}=\sqrt[6]{\mathbf{3}}$

e) $\frac{\sqrt{2}}{\sqrt[4]{2^2}}=\frac{2^{1/2}}{2^{2/4}}=\mathbf{1}$

f) $\sqrt{2\cdot\sqrt{2\cdot\sqrt{2}}}=\left(2\cdot\left(2\cdot 2^{1/2}\right)^{1/2}\right)^{1/2}=2^{1/2}\cdot 2^{1/4}\cdot 2^{1/8}=2^{1/2+1/4+1/8}=2^{7/8}=\sqrt[8]{2^7}$

Exercice 9.17: Il s'agit de proposer une chaîne d'égalités passant du terme de gauche à celui de droite. Mais… Ne regardez pas trop vite ci-dessous…

a) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n]{a^{1/m}} = \left(a^{1/m}\right)^{1/n} = a^{\frac{1}{m}\cdot\frac{1}{n}} = a^{\frac{1}{n\cdot m}} = \sqrt[n\cdot m]{a}$

b) $\sqrt[m\cdot n]{a^{m\cdot p}} = \left(a^{m\cdot p}\right)^{\frac{1}{m\cdot n}} = a^{\frac{m\cdot p}{m\cdot n}} = a^{\frac{p}{n}} = \left(a^{p}\right)^{\frac{1}{n}} = \sqrt[n]{a^{p}}$

c) $\sqrt[6]{2}$ **d)** $\sqrt[2]{2^3} = 2\sqrt{2}$ **e)** 6

Exercice 9.18: Vous avez dû obtenir un nombre proche de 2. Voici pourquoi il s'agit même exactement de 2.

1) Partons de $2 = \sqrt{4}$ que l'on peut écrire: $2 = \sqrt{2+\mathbf{2}}$

2) Substituons le dernier 2 grâce à cette même dernière égalité. On obtient:

$$2 = \sqrt{2+\sqrt{2+\mathbf{2}}}$$

3) Réitérerons ce même procédé:

$$2 = \sqrt{2+\sqrt{2+\sqrt{2+\mathbf{2}}}}$$

4) Et encore une fois:

$$2 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\mathbf{2}}}}}$$

5) Et ainsi de suite:

$$2 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}$$

Je vous laisse trouver l'imbrication de racines qui donne 3 à la place de 2.

Exercice 9.19: Exercice BONUS.