Corrigé chapitre 4 Probabilités

Ex. 4.1

a) $\frac{1}{6} = 16,67\%$ b) $\frac{3}{6} = 50\% = \frac{1}{2}$ c) $\frac{2}{6} = 33,33\% = \frac{1}{3}$

Ex. 4.2

a) $\frac{4}{36} = \frac{1}{9} = 11,11\%$ b) $\frac{9}{36} = \frac{1}{4} = 25\%$ c) $\frac{1}{36} = 2,78\%$

Ex. 4.3

$|U| = C_3^{36} = 7140$

a) $\frac{C_3^4}{C_3^{36}} = \frac{4}{7140} = \frac{1}{1785} = 0,06\%$

b) $\frac{C_2^4 \cdot C_1^4}{C_3^{36}} = \frac{24}{7140} = \frac{2}{595} = 0,34\%$

c) $\frac{C_1^4 \cdot C_2^{32} + C_2^4 \cdot C_1^{32} + C_3^4}{C_3^{36}} = \frac{2180}{7140} = \frac{109}{357} = 30,53\%$

ou $1 - P(0 \text{ valet}) = 1 - \frac{C_3^{32}}{C_3^{36}} = 1 - \frac{4960}{7140} = \frac{2180}{7140} = \frac{109}{357} = 30,53\%$

Ex. 4.4

$|U| = 2^4 = 16$

a) $\frac{1}{2^4} = \frac{1}{16} = 6,25\%$

b) $\frac{C_2^4}{2^4} = \frac{6}{16} = \frac{3}{8} = 37,5\%$

c) $P(0\text{ pile}) + P(1\text{ pile}) = \frac{1}{2^4} + \frac{C_1^4}{2^4} = \frac{5}{16} = 31,25\%$

Ex. 4.5

$|\Omega| = 6^2 = 36$

a) $\frac{6}{6^2} = \frac{6}{36} = \frac{1}{6} = 16,67\%$

b) $\frac{2}{6^2} = \frac{2}{36} = \frac{1}{18} = 5,56\%$

c) $\frac{1}{6^2} = \frac{1}{36} = 2,78\%$

d) $\Sigma = 7 \rightarrow 6$ possibilités $\Rightarrow \frac{6}{6^2} = \frac{6}{36} = \frac{1}{6} = 16,67\%$

e) $P(\Sigma = 2) + P(\Sigma = 3) = \frac{1}{6^2} + \frac{2}{6^2} = \frac{3}{36} = \frac{1}{12} = 8,33\%$

f) $1 - P(\Sigma = 12) = 1 - \frac{1}{6^2} = \frac{35}{36} = 97,22\%$

Ex. 4.6

$|\Omega| = C^{52}_{13} \Rightarrow P = \frac{C^4_3 \cdot C^{48}_{10}}{C^{52}_{13}} = 4,12\% = \frac{858}{20'825}$

Ex. 4.7

$|\Omega| = C^{36}_3 = 7140$

a) $\frac{C^9_3 \cdot 4}{C^{36}_3} = \frac{336}{7140} = \frac{4}{85} = 4,71\%$

b) $\frac{C^4_3}{C^{36}_3} = \frac{4}{7140} = \frac{1}{1785} = 0,06\%$

c) $\frac{C^4_1 \cdot C^4_2}{C^{36}_3} = \frac{24}{7140} = \frac{2}{595} = 0,34\%$

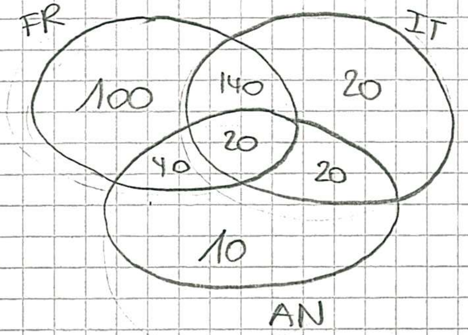
d) $\frac{C^4_1 \cdot C^9_2 \cdot C^{27}_1}{C^{36}_3} = \frac{3888}{7140} = \frac{324}{595} = 54,45\%$

e) $\frac{C^{18}_2 \cdot C^{18}_1}{C^{36}_3} = \frac{2754}{7140} = \frac{27}{70} = 38,57\%$

f) $\frac{C^4_1 \cdot C^4_1 \cdot C^4_1}{C^{36}_3} = \frac{64}{7140} = \frac{16}{1785} = 0,90\%$

g) $\frac{C^9_1 \cdot C^9_1 \cdot C^9_1}{C^{36}_3} = \frac{729}{7140} = \frac{243}{2380} = 10,21\%$

Ex. 4.8



a) $\dfrac{140+40+20}{500} = \dfrac{200}{500} = \dfrac{2}{5} = 40\%$

b) $\dfrac{100+140+20+40+20+20+10}{500} = \dfrac{350}{500}$

$= \dfrac{7}{10} = 70\%$

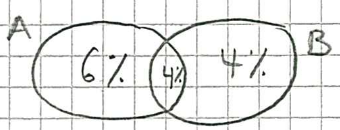
Ex. 4.9

$|U| = 543 + 310 + 156 + 81 = 1090$

a) $\dfrac{310}{1090} = 28{,}44\%$

b) $P(0 \text{ fois}) + P(1 \text{ fois}) = \dfrac{543+310}{1090} = \dfrac{853}{1090} = 78{,}26\%$

Ex. 4.10



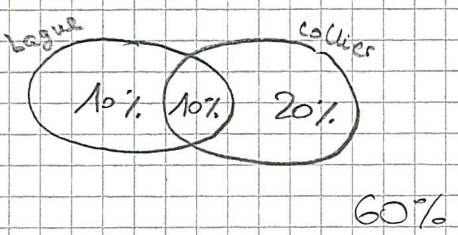
a) $6 + 4 + 4 = 14\%$

b) 6%

c) $6 + 4 = 10\%$

d) $100 - 6 - 4 - 4 = 86\%$

Ex. 4.11



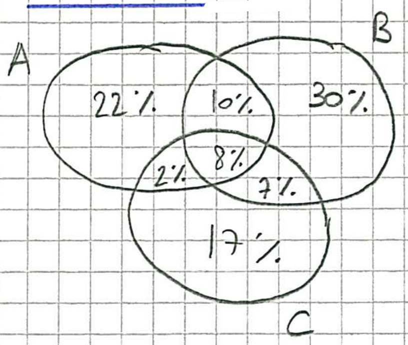
60% n'ont rien ⇒ 40% portent quelque chose

20% + 30% = 50% ⇒ 10% portent les 2

a) $10 + 10 + 20 = 40\%$

b) 10%

Ex. 4.12



a) $22 + 10 + 30 + 2 + 8 + 7 + 17 = 96\%$

b) $100 - 96 = 4\%$

c) $10 + 2 + 7 = 19\%$

d) 22%

Ex. 4.13

$P(A) = P(B) = 2 \cdot P(C)$

$\Rightarrow P(A) = \frac{2}{5} = 40\%$ $P(B) = \frac{2}{5} = 40\%$ $P(C) = \frac{1}{5} = 20\%$

Ex. 4.14

$P(1) = 10\%$, $P(6) = 40\%$ $\Rightarrow$ Il reste 50% pour les 4 autres faces

$\Rightarrow P(2) = P(3) = P(4) = P(5) = 12,5\%$

a) $12,5\% = \frac{1}{8}$ b) $10 + 12,5 + 12,5 = 35\% = \frac{7}{20}$

c) $12,5 + 35 = 47,5\% = \frac{19}{40}$

Ex. 4.15

$P(A) = \frac{5}{36} = 13,89\%$ $\qquad P(B) = \frac{6 \cdot 5}{36} = \frac{30}{36} = 83,33\%$ $\qquad P(C) = \frac{3 \cdot 6}{36} = \frac{18}{36} = 50\%$

$$P(A|B) = \frac{|A \text{ et } B|}{|B|} = \frac{4}{30} = \frac{2}{15} = 13,33\%$$

$$P(A|C) = \frac{|A \text{ et } C|}{|C|} = \frac{2}{18} = \frac{1}{9} = 11,11\%$$

$$P(A|\overline{B}) = \frac{|A \text{ et pas } B|}{|\text{pas } B|} = \frac{1}{6} = 16,67\%$$

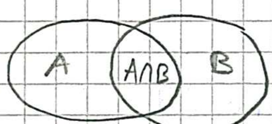
$$P(A|\overline{C}) = \frac{|A \text{ et pas } C|}{|\text{pas } C|} = \frac{3}{3 \cdot 6} = \frac{3}{18} = \frac{1}{6} = 16,67\%$$

Ex. 4.16

- $P(B|A) = \frac{|B \text{ et } A|}{|A|} = \frac{1}{9} = 11,11\%$
- $P(A|C) = \frac{|A \text{ et } C|}{|C|} = \frac{9}{12} = \frac{3}{4} = 75\%$
- $P(B|C) = \frac{|B \text{ et } C|}{|C|} = \frac{1}{12} = 8,33\%$
- $P(C|B) = \frac{|C \text{ et } B|}{|B|} = \frac{1}{1} = 1 = 100\%$

Ex. 4.17

$A \cup B$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B) \quad \Rightarrow \quad P(A \cap B) = \frac{1}{4}$$

- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{5/8} = \frac{1}{4} \cdot \frac{8}{5} = \frac{2}{5}$

- $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/4}{3/8} = \frac{1}{4} \cdot \frac{8}{3} = \frac{2}{3}$

Ex. 4.18

$|U| = A_4^{36} = 1'413'720$

a) $\dfrac{1}{A_4^{36}} = 0{,}000071\,\%$

b) $\dfrac{4!}{A_4^{36}} = 0{,}0017\,\%$

c) $P(4\text{ as} \mid 2\text{ premières as}) = \dfrac{|4\text{ as et } 2\text{ premières as}|}{|2\text{ premières as}|} = \dfrac{4!}{4\cdot 3\cdot 32\cdot 31} =$

$= \dfrac{24}{11904} = 0{,}20\,\%$

d) $\dfrac{A_1^4 \cdot A_3^{32} \cdot 4}{A_4^{36}} = \dfrac{476160}{1'413'720} = 33{,}68\,\%$

e) $1 - P(0\text{ as}) = 1 - \dfrac{A_4^{32}}{A_4^{36}} = 1 - \dfrac{863040}{1413720} = \dfrac{550680}{1413720} = 38{,}95\,\%$

f) $P(\text{au moins } 1\text{ as} \mid \text{première pas un as}) = \dfrac{|\text{au moins } 1\text{ as et première pas un as}|}{|\text{première pas un as}|}$

$= \dfrac{A_1^4 \cdot A_3^{32} \cdot 3 + A_2^4 \cdot A_2^{32} \cdot 3 + A_1^{32} \cdot A_3^4}{32\cdot 35\cdot 34\cdot 33} = \dfrac{392960}{1256640} = 31{,}28\,\%$

Ex. 4.19

$|U| = A_4^8 = 1680$

a) $\dfrac{4!}{A_4^8} = \dfrac{24}{1680} = 1{,}43\,\%$

b) $1 - P(0\text{ as}) = 1 - \dfrac{4!}{A_4^8} = \dfrac{1656}{1680} = 98{,}57\,\%$

c) $\dfrac{4!}{A_4^8} = \dfrac{24}{1680} = 1{,}43\,\%$

d) $\dfrac{2^4 \cdot 4!}{A_4^8} = \dfrac{384}{1680} = 22{,}86\,\%$

Ex. 4.19 (suite)

e) $P(4\text{ as} \mid 1^{\text{ère}}\text{ as}) = \dfrac{|4\text{ as et } 1^{\text{ère}}\text{ as}|}{|1^{\text{ère}}\text{ as}|} = \dfrac{4!}{4\cdot 7\cdot 6\cdot 5} = \dfrac{24}{840} = 2,86\,\%$

f) $P(4\text{ as} \mid 1^{\text{ère}}\text{ as rouge}) = \dfrac{|4\text{ as et } 1^{\text{ère}}\text{ as rouge}|}{|1^{\text{ère}}\text{ as rouge}|} = \dfrac{2\cdot 3\cdot 2\cdot 1}{2\cdot 7\cdot 6\cdot 5} = \dfrac{12}{420} = 2,86\,\%$

g) $P(4\text{ as} \mid 1^{\text{ère}}\text{ as } \heartsuit) = \dfrac{|4\text{ as et } 1^{\text{ère}}\text{ as } \heartsuit|}{|1^{\text{ère}}\text{ as } \heartsuit|} = \dfrac{1\cdot 3\cdot 2\cdot 1}{1\cdot 7\cdot 6\cdot 5} = \dfrac{6}{210} = 2,86\,\%$

Ex. 4.20

$|\Omega| = C_2^8 = 28$

a) $\dfrac{C_2^4}{C_2^8} = \dfrac{6}{28} = \dfrac{3}{14} = 21,43\,\%$

b) $\dfrac{C_2^2}{C_2^8} = \dfrac{1}{28} = 3,57\,\%$

c) $\dfrac{C_1^4 \cdot C_1^4 + C_2^4}{C_2^8} = \dfrac{22}{28} = \dfrac{11}{14} = 78,57\,\%$

d) i) $P(2\text{ as} \mid 1\text{ as}) = \dfrac{|2\text{ as}|}{|\text{au moins } 1\text{ as}|} = \dfrac{C_2^4}{C_1^4 \cdot C_1^4 + C_2^4} = \dfrac{6}{22} = \dfrac{3}{11} = 27,27\,\%$

ii) $P(2\text{ as} \mid 1\text{ as rouge}) = \dfrac{|2\text{ as dont au moins } 1\text{ rouge}|}{|\text{au moins } 1\text{ rouge}|} = \dfrac{C_1^2 \cdot C_1^2 + C_2^2}{C_1^2 \cdot C_1^6 + C_2^2}$

$= \dfrac{5}{13} = 38,46\,\%$

iii) $P(2\text{ as} \mid \text{as } \heartsuit) = \dfrac{|2\text{ as dont as } \heartsuit|}{|\text{as } \heartsuit|} = \dfrac{C_1^1 \cdot C_1^3}{C_1^1 \cdot C_1^7} = \dfrac{3}{7} = 42,86\,\%$

Ex. 4.21

$P(+\text{ de } 20'000 \mid +\text{ de } 10'000) = \dfrac{P(+\text{ de } 20'000)}{P(+\text{ de } 10'000)} = \dfrac{0,4}{0,8} = 0,5 = 50\,\%$

Ex. 4.22

$\Sigma = 10$: 3 possibilités $\quad$ $\Sigma = 11$: 2 possibilités $\quad$ $\Sigma = 12$: 1 possibilité

$5+5 \quad 4+6 \quad 6+4 \qquad 6+5 \quad 5+6 \qquad 6+6$

a) $$P(\Sigma > 9 \mid 1^{er} \text{ dé} = 5) = \frac{|\Sigma > 9 \text{ et } 1^{er} \text{ dé} = 5|}{|1^{er} \text{ dé} = 5|} = \frac{2}{1 \cdot 6} = \frac{2}{6} = \frac{1}{3} = 33,33\,\%$$

b) $$P(\Sigma > 9 \mid 1 \text{ fois le } 5 \text{ au moins}) = \frac{|\Sigma > 9 \text{ et au moins } 1 \text{ fois le } 5|}{|\text{au moins } 1 \text{ fois le } 5|}$$

$$= \frac{3}{11} = 27,27\,\%$$

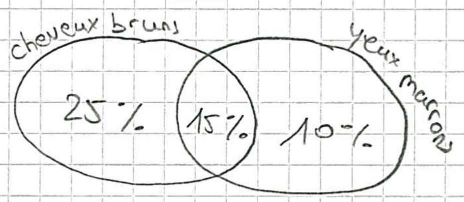
c) $$P(\Sigma = 6 \mid 2 \text{ chiffres différents}) = \frac{|\Sigma = 6 \text{ et } 2 \text{ chiffres différents}|}{|2 \text{ chiffres différents}|}$$

$$= \frac{4}{6 \cdot 5} = \frac{4}{30} = \frac{2}{15} = 13,33\,\%$$

d) $$P(\Sigma < 5 \mid 2 \text{ chiffres différents}) = \frac{|\Sigma < 5 \text{ et } 2 \text{ chiffres différents}|}{|2 \text{ chiffres différents}|}$$

$$= \frac{\underset{\Sigma=3}{2} + \underset{\Sigma=4}{2}}{6 \cdot 5} = \frac{4}{30} = \frac{2}{15} = 13,33\,\%$$

Ex. 4.23



a) $P(\text{yeux marrons} \mid \text{cheveux bruns})$

$$= \frac{P(\text{yeux marrons et cheveux bruns})}{P(\text{cheveux bruns})} = \frac{0,15}{0,4}$$

$$= 0,375 = 37,5\,\% = \frac{3}{8}$$

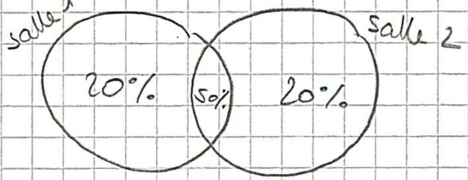
b) $$P(\text{cheveux pas bruns} \mid \text{yeux marrons}) = \frac{P(\text{cheveux pas bruns et yeux marrons})}{P(\text{yeux marrons})}$$

$$= \frac{0,1}{0,25} = 0,4 = 40\,\% = \frac{2}{5}$$

c) $100 - 25 - 15 - 10 = 50\,\% = \frac{1}{2}$

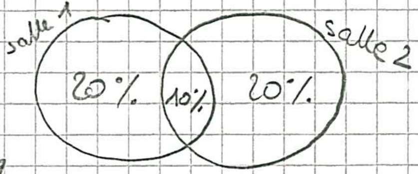
Ex. 4.24

Salles occupées



2 salles libres : 10%

Salles libres



2 salles occupées : 50%

a) $20 + 10 = 30\%$

b) 10%

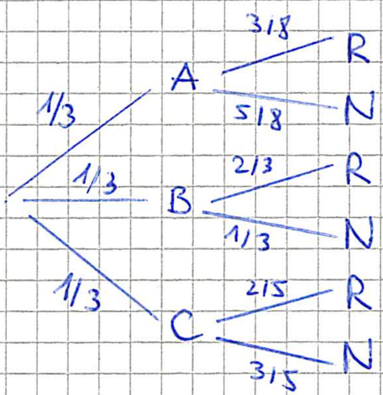
c) $20 + 10 + 20 = 50\%$

d) $20 + 20 = 40\%$

e) $P(2^e \text{ libre} \mid 1^e \text{ occupée}) = \dfrac{P(1^{ère} \text{ occupée et } 2^{ème} \text{ libre})}{P(1^{ère} \text{ occupée})} = \dfrac{0,2}{0,2+0,5}$

$= \dfrac{0,2}{0,7} = 28,57\% = \dfrac{2}{7}$

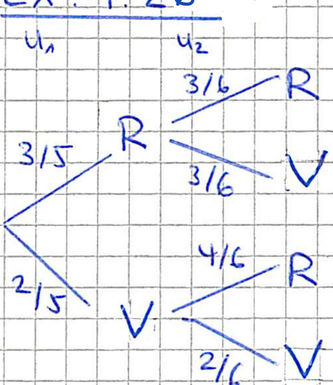
Ex. 4.25



a) $P(R) = \dfrac{1}{3}\cdot\dfrac{3}{8} + \dfrac{1}{3}\cdot\dfrac{2}{3} + \dfrac{1}{3}\cdot\dfrac{2}{5} = \dfrac{173}{360} = 48,06\%$

b) $P(A \mid R) = \dfrac{P(A \text{ et } R)}{P(R)} \underset{(a)}{=} \dfrac{\frac{1}{3}\cdot\frac{3}{8}}{\frac{173}{360}} = \dfrac{45}{173} = 26,01\%$

Ex. 4.26



a) $P(2^e R) = \dfrac{3}{5}\cdot\dfrac{3}{6} + \dfrac{2}{5}\cdot\dfrac{4}{6} = \dfrac{17}{30} = 56,67\%$

b) $P(2^e R \mid 1^e R) = \dfrac{P(1^e \text{ et } 2^e R)}{P(1^e R)} = \dfrac{\frac{3}{5}\cdot\frac{3}{6}}{\frac{3}{5}\cdot\frac{3}{6}+\frac{3}{5}\cdot\frac{3}{6}} = \dfrac{1}{2} = 50\%$

(Peut être raisonné par logique !)

c) $P(1^e R \mid 2^e R) = \dfrac{P(1^e \text{ et } 2^e R)}{P(2^e R)} \underset{(a)}{=} \dfrac{\frac{3}{5}\cdot\frac{3}{6}}{\frac{17}{30}} = \dfrac{9}{17}$

$= 52,94\%$

Ex. 4.27

$\boxed{A}$ $|\mathcal{U}| = A^{15}_3 = 2730$

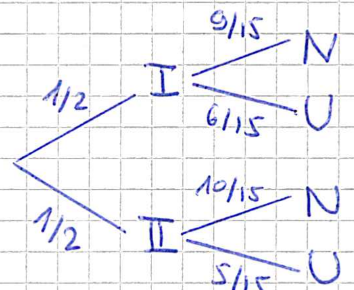
(a) $\dfrac{A^9_3}{A^{15}_3} = \dfrac{504}{2730} = \dfrac{12}{65} = 18,46\%$

(b) $1 - P(0 \text{ usagée}) = 1 - \dfrac{A^9_3}{A^{15}_3} = \dfrac{53}{65} = 81,54\%$

(c) $P(3^e \text{ usagée} \mid 1^e \text{ et } 2^e \text{ usagées}) = \dfrac{|3 \text{ usagées}|}{|1^e \text{ et } 2^e \text{ usagées}|} = \dfrac{A^6_3}{6 \cdot 5 \cdot 13} = \dfrac{120}{390}$

$= \dfrac{4}{13} = 30,77\%$

$\boxed{B}$ (a)

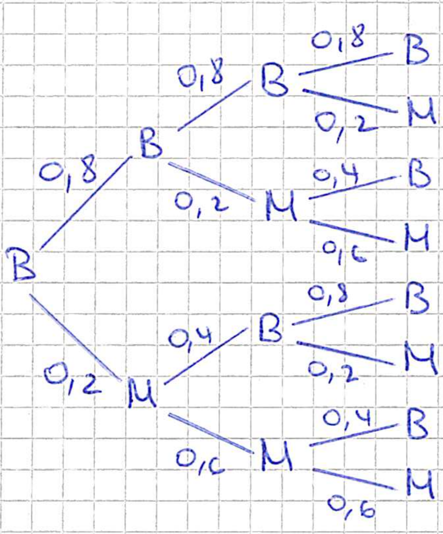


(b) $P(N) = \dfrac{1}{2} \cdot \dfrac{9}{15} + \dfrac{1}{2} \cdot \dfrac{10}{15} = \dfrac{19}{30} = 63,33\%$

(c) $P(\text{II} \mid U) = \dfrac{P(\text{II et } U)}{P(U)}$

$= \dfrac{\frac{1}{2} \cdot \frac{5}{15}}{\frac{1}{2} \cdot \frac{6}{15} + \frac{1}{2} \cdot \frac{5}{15}} = \dfrac{\frac{5}{30}}{\frac{11}{30}} = \dfrac{5}{11} = 45,45\%$

Ex. 4.28

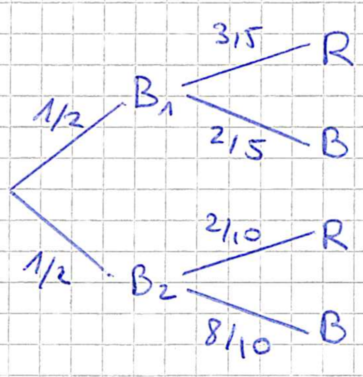


a) $0,8 \cdot 0,8 \cdot 0,8 = 51,2\%$

b) $0,8 \cdot 0,8 \cdot 0,8 + 0,8 \cdot 0,2 \cdot 0,4 + 0,2 \cdot 0,4 \cdot 0,8$
$+ 0,2 \cdot 0,6 \cdot 0,4$

$= 68,8\%$

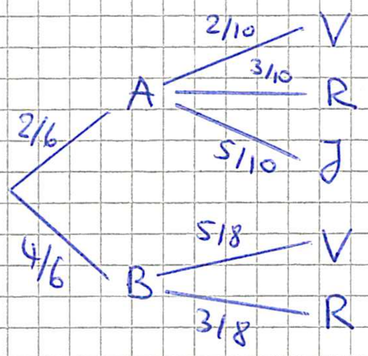
Ex. 4.29



$P(B_1 \mid R) = \dfrac{P(B_1 \text{ et } R)}{P(R)} = \dfrac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{2}{10}} = \dfrac{\frac{3}{10}}{\frac{2}{5}}$

$= \dfrac{3}{4} = 75\%$

Ex. 4.30



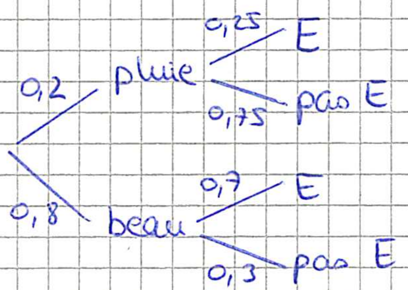
a) $P(V) = \frac{2}{6} \cdot \frac{2}{10} + \frac{4}{6} \cdot \frac{5}{8} = \frac{29}{60} = 48,33\,\%$

b) $P(V \mid >2) = P(V \mid B) = \left(\frac{P(V \text{ et } B)}{P(B)} = \frac{\frac{4}{6} \cdot \frac{5}{8}}{\frac{4}{6}} \right)$
$= \frac{5}{8} = 62,5\,\%$

c) $P(<3 \mid R) = P(A \mid R) = \frac{P(A \text{ et } R)}{P(R)} = \frac{\frac{2}{6} \cdot \frac{3}{10}}{\frac{2}{6} \cdot \frac{3}{10} + \frac{4}{6} \cdot \frac{3}{8}}$
$= \frac{\frac{1}{10}}{\frac{7}{20}} = \frac{2}{7} = 28,57\,\%$

d) $P(>2 \mid J) = P(B \mid J) = 0\,\%$

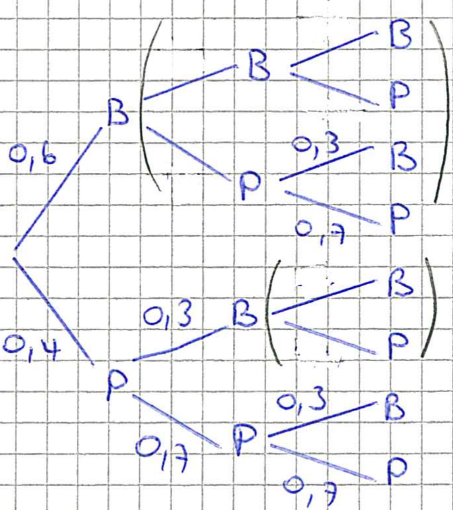
Ex. 4.31



a) $P(E) = 0,2 \cdot 0,25 + 0,8 \cdot 0,7 = 61\,\%$

b) $P(\text{beau} \mid E) = \frac{P(\text{beau et } E)}{P(E)} \underset{(a)}{=} \frac{0,8 \cdot 0,7}{0,61}$
$= 91,80\,\%$

Ex. 4.32



a) annulé si 3 fois pluie $\Rightarrow 0,4 \cdot 0,7 \cdot 0,7 = 19,6\,\%$

b) $0,6 + 0,4 \cdot 0,3 = 72\,\%$

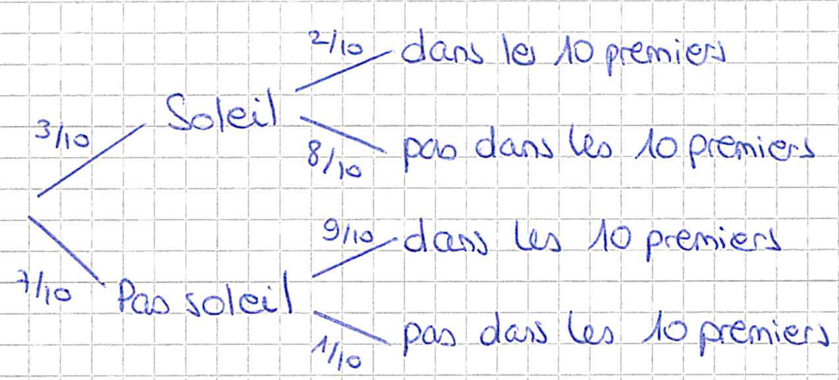
c) $P(1^{e} \text{ jour} \mid \text{course a lieu}) = \frac{P(\text{course } 1^{e} \text{ jour})}{P(\text{course a lieu})}$
$= \frac{0,6}{1 - 0,196} = 74,63\,\%$

d) $0,6^3 \cdot 0,4^2 \cdot C_3^5 = 34,56\,\%$

e) $1 - P(\underset{5 \text{ fois}}{\text{pas annulée}}) = 1 - (0,804)^5 = 66,40\,\%$

car $P(\text{annulée}) = 0,196 \Rightarrow P(\text{pas annulée}) = 0,804$

Ex. 4.33



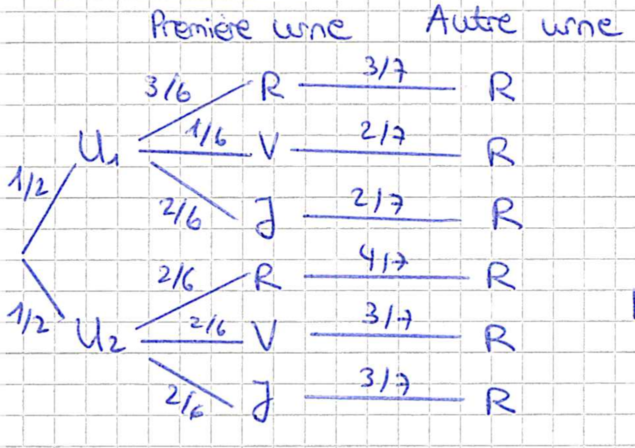
$P(\text{pas soleil} \mid 7^e)$

$$= \frac{P(\text{pas soleil et dans les 10 premiers})}{P(\text{dans les 10 premiers})}$$

$$= \frac{\frac{7}{10}\cdot\frac{9}{10}}{\frac{3}{10}\cdot\frac{2}{10}+\frac{7}{10}\cdot\frac{9}{10}} = \frac{\frac{63}{100}}{\frac{69}{100}} = \frac{63}{69} = \frac{21}{23}$$

$= 91{,}30\%$

Ex. 4.34



a) $P(R) = \frac{1}{2}\cdot\frac{3}{6}\cdot\frac{3}{7} + \frac{1}{2}\cdot\frac{1}{6}\cdot\frac{2}{7} + \frac{1}{2}\cdot\frac{2}{6}\cdot\frac{2}{7} + \frac{1}{2}\cdot\frac{2}{6}\cdot\frac{4}{7} + \frac{1}{2}\cdot\frac{2}{6}\cdot\frac{3}{7}$

$+ \frac{1}{2}\cdot\frac{2}{6}\cdot\frac{3}{7} = \frac{5}{12} = 41{,}67\%$

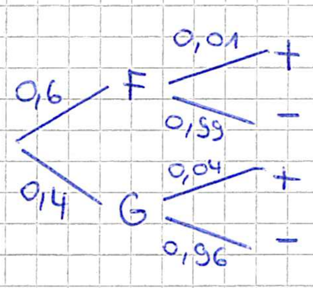
b) $P(R \mid 1^e R) = \frac{P(2R)}{P(1^e R)} = \frac{\frac{1}{2}\cdot\frac{3}{6}\cdot\frac{3}{7}+\frac{1}{2}\cdot\frac{2}{6}\cdot\frac{4}{7}}{\frac{1}{2}\cdot\frac{3}{6}+\frac{1}{2}\cdot\frac{2}{6}}$

$= \frac{\frac{17}{84}}{\frac{5}{12}} = \frac{17}{35} = 48{,}57\%$

c) $P(R \mid U_1) = \frac{P(U_1 \text{ et } R)}{P(U_1)} = \frac{\frac{1}{2}\cdot\frac{3}{6}\cdot\frac{3}{7}+\frac{1}{2}\cdot\frac{1}{6}\cdot\frac{2}{7}+\frac{1}{2}\cdot\frac{2}{6}\cdot\frac{2}{7}}{\frac{1}{2}} = \frac{\frac{5}{28}}{\frac{1}{2}} = \frac{5}{14} = 35{,}71\%$

d) $P(U_1 \mid R) = \frac{P(U_1 \text{ et } R)}{P(R)} = \frac{\overset{(c)\to}{\frac{5}{28}}}{\underset{(a)\to}{\frac{5}{12}}} = \frac{3}{7} = 42{,}86\%$

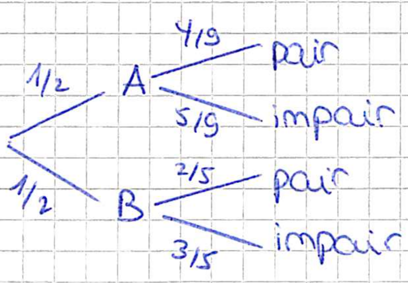
Ex. 4.35



$P(F \mid +) = \frac{P(F \text{ et } +)}{P(+)} = \frac{0{,}6\cdot 0{,}01}{0{,}6\cdot 0{,}01 + 0{,}4\cdot 0{,}04}$

$= \frac{3}{11} = 27{,}27\%$

Ex. 4.36



$P(A \mid \text{pair}) = \frac{P(A \text{ et pair})}{P(\text{pair})} = \frac{\frac{1}{2}\cdot\frac{4}{9}}{\frac{1}{2}\cdot\frac{4}{9}+\frac{1}{2}\cdot\frac{2}{5}} = \frac{\frac{2}{9}}{\frac{19}{45}} =$

$= \frac{10}{19} = 52{,}63\%$

Ex. 4.37

$$P = \underbrace{\frac{1}{6}\cdot\frac{1}{4}\cdot\frac{2}{3}}_{3^e\text{ échoue}} + \underbrace{\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{1}{3}}_{2^e\text{ échoue}} + \underbrace{\frac{5}{6}\cdot\frac{1}{4}\cdot\frac{1}{3}}_{1^e\text{ échoue}} + \underbrace{\frac{1}{6}\cdot\frac{1}{4}\cdot\frac{1}{3}}_{\text{les 3 réussissent}} = \frac{11}{72} = 15,28\,\%$$

Ex. 4.38

1) (a) $$\frac{2\cdot 4}{6\cdot 6} = \frac{8}{36} = \frac{2}{9} = 22,22\,\%$$

(b) $$\frac{\overset{P}{2}\cdot\overset{P}{4} + \overset{P}{2}\cdot\overset{I}{2} + \overset{I}{4}\cdot\overset{P}{4}}{6\cdot 6} = \frac{28}{36} = \frac{7}{9} = 77,78\,\%$$

(c) $$\frac{2\cdot 2 + 4\cdot 4}{6\cdot 6} = \frac{20}{36} = \frac{5}{9} = 55,56\,\%$$

2) (a) $$\underset{\text{s'il obtient } 1}{0} + \underset{3}{\frac{1}{36}} + \underset{7}{\frac{4}{36}} + \underset{8}{\frac{4}{36}} + \underset{10}{\frac{5}{36}} + \underset{11}{\frac{5}{36}} = \frac{19}{36} = 52,78\,\%$$

(b) $$P(10 \mid \text{gagné}) = \frac{P(10 \text{ et gagné})}{P(\text{gagné})} \underset{(a)}{=} \frac{\frac{5}{36}}{\frac{19}{36}} = \frac{5}{19} = 26,32\,\%$$

Ex. 4.39

a) $P(X=1) = \frac{3}{5} = 60\%$

$P(X=2) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} = 30\%$ Total = 100%

$P(X=3) = \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{3}{3} = \frac{6}{60} = 10\%$

$\left.\begin{array}{l} P(X=4) = 0\% \\ P(X=5) = 0\% \end{array}\right\}$ on a forcément tiré une boule verte avant

b) $E(X) = 1 \cdot 0{,}6 + 2 \cdot 0{,}3 + 3 \cdot 0{,}1 + 4 \cdot 0 + 5 \cdot 0 = 1{,}5$

Si l'on répète l'expérience un grand nombre de fois, il faudra en moyenne <u>1,5</u> tirages pour obtenir une boule verte (soit <u>1</u> ou <u>2</u> tirages)

Ex. 4.40

a) $P(X=0) = \frac{C_9^{32}}{C_9^{36}} = 29{,}79\%$

$P(X=1) = \frac{C_1^4 \cdot C_8^{32}}{C_9^{36}} = 44{,}69\%$

$P(X=2) = \frac{C_2^4 \cdot C_7^{32}}{C_9^{36}} = 21{,}45\%$

$P(X=3) = \frac{C_3^4 \cdot C_6^{32}}{C_9^{36}} = 3{,}85\%$

$P(X=4) = \frac{C_4^4 \cdot C_5^{32}}{C_9^{36}} = 0{,}21\%$

b) $E[X] = 0 \cdot 0{,}2979 + 1 \cdot 0{,}4469 + 2 \cdot 0{,}2145 + 3 \cdot 0{,}0385 + 4 \cdot 0{,}0021$

$= 0{,}9998 \cong 1$ (sans arrondir les probas $\Rightarrow = 1$)

Si l'on répète l'expérience un grand nombre de fois, on aura en moyenne 1 as.

c) Faux : On peut en tirer 0, 1, 2, 3 ou 4

Ex. 4.41

a) $P(X=1) = \frac{1}{36}$ (1 et 1)

$P(X=2) = \frac{3}{36} = \frac{1}{12}$ (1 et 2 / 2 et 1 / 2 et 2)

$P(X=3) = \frac{5}{36}$ (1 et 3 / 2 et 3 / 3 et 3 / 3 et 2 / 3 et 1)

$P(X=4) = \frac{7}{36}$

$P(X=5) = \frac{9}{36} = \frac{1}{4}$

$P(X=6) = \frac{11}{36}$

b) $E[X] = 1 \cdot \frac{1}{36} + 2 \cdot \frac{1}{12} + 3 \cdot \frac{5}{36} + 4 \cdot \frac{7}{36} + 5 \cdot \frac{1}{4} + 6 \cdot \frac{11}{36} = \frac{161}{36} \cong 4,47$

Si l'on répète l'expérience un grand nombre de fois, on aura en moyenne un résultat de 4,5.

c) Faux : si on lance une seule fois les deux dés, on a plus de chances d'avoir le 6.

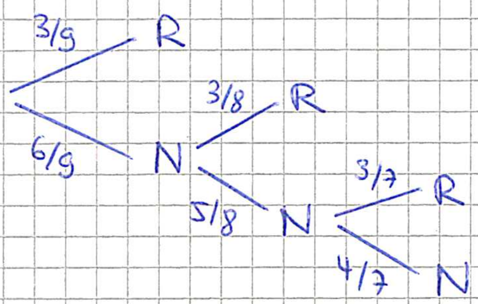
Ex. 4.42

a) $P(X=10) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $= 25\%$ $P(X=5) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $= 25\%$ $P(X=-15) = \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = \frac{1}{2}$ $= 50\%$

b) $E[X] = 10 \cdot 0,25 + 5 \cdot 0,25 + (-15) \cdot 0,5 = -3,75$

c) Non, car si l'on y joue un grand nombre de fois, on perdra en moyenne 3,75 francs.

Ex. 4.43



Gain/perte	
+100.–	$P(X=100) = \frac{3}{9} = \frac{1}{3}$
+50.–	$P(X=50) = \frac{6}{9}\cdot\frac{3}{8} = \frac{1}{4}$
+0.–	$P(X=0) = \frac{6}{9}\cdot\frac{5}{8}\cdot\frac{3}{7} = \frac{5}{28}$
-150.–	$P(X=-150) = \frac{6}{9}\cdot\frac{5}{8}\cdot\frac{4}{7} = \frac{5}{21}$

$$E[X] = 100\cdot\frac{1}{3} + 50\cdot\frac{1}{4} + 0\cdot\frac{5}{28} - 150\cdot\frac{5}{21} = \frac{425}{42} \cong 10,12$$

$\Rightarrow$ oui, on accepte car on gagne en moyenne un peu plus de 10 francs par partie (si l'on joue un grand nombre de fois !)

Ex. 4.44

♡ : gain net de 2.– $\Rightarrow P(X=2) = \frac{9}{36} = \frac{1}{4}$

◇ : gain net de 1.– $\Rightarrow P(X=1) = \frac{9}{36} = \frac{1}{4}$

♠ ou ♣ : perte de 3.– $\Rightarrow P(X=-3) = \frac{18}{36} = \frac{1}{2}$

$$E[X] = 2\cdot\frac{1}{4} + 1\cdot\frac{1}{4} - 3\cdot\frac{1}{2} = -0,75$$

À ce jeu, on perd en moyenne 0,75 francs par partie (si l'on joue un grand nombre de fois)

Ex. 4.45

$P(X=3) = \frac{1}{4} = 25\%$ $\quad P(X=1) = \frac{1}{2} = 50\%$ $\quad P(X=-k) = \frac{1}{4} = 25\%$

$$E[X] = 3\cdot 0,25 + 1\cdot 0,5 + (-k)\cdot 0,25 = 0$$

$$1,25 - 0,25k = 0$$

$$-0,25k = -1,25$$

$$k = 5$$

Il faut que $k=5$ pour avoir un jeu équitable (tq $E(X)=0$)

Ex. 4.46

X = gain net

a) $P(X=6) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} = \frac{15}{36}$ $\qquad P(X=-6) = \frac{21}{36}$

dé blanc = 2 3 4 5 6

$$E[X] = 6 \cdot \frac{15}{36} - 6 \cdot \frac{21}{36} = -1$$

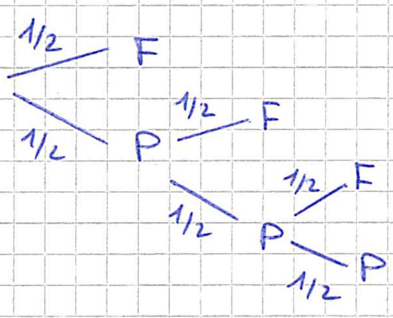
$\Rightarrow$ le jeu n'est pas équitable (on perd en moyenne 1 francs par partie)

b) Il faudrait payer 5.– par partie

(En effet, $E(X) = 7 \cdot \frac{15}{36} - 5 \cdot \frac{21}{36} = 0$)

Ex. 4.47

X = gain



Gain/perte	
+ 8.–	$P(X=8) = \frac{1}{2} = 50\%$
+ 5.–	$P(X=5) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 25\%$
+ 2.–	$P(X=2) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = 12{,}5\%$
– 40.–	$P(X=-40) = \frac{1}{8} = 12{,}5\%$

$$E[X] = 8 \cdot 0{,}5 + 5 \cdot 0{,}25 + 2 \cdot 0{,}125 - 40 \cdot 0{,}125 = 0{,}5$$

En moyenne, on gagne 50 centimes par partie.

Ex. 4.48

X = tarif d'entrée

$P(X=i) = \frac{1}{6}$ pour $i = 0, 5, 10, 15, 20, 25$

$$E[X] = 0 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 10 \cdot \frac{1}{6} + 15 \cdot \frac{1}{6} + 20 \cdot \frac{1}{6} + 25 \cdot \frac{1}{6} = 12{,}5$$

En moyenne, le joueur paie 12,5 francs.
Il vaut donc mieux choisir le tarif classique.