1.2

1) $$\begin{cases} x+2y+z=8 \\ 2x+y-z=1 \\ 3x-y+2z=7 \end{cases} \overset{\substack{L_2 \to L_2-2L_1 \\ L_3 \to L_2-3L_1}}{\Longrightarrow} \begin{cases} x+2y+z=8 \\ -3y-3z=-15 \\ -7y-z=-17 \end{cases} \overset{L_2 \to -1/3\,L_2}{\Longrightarrow}$$

$$\begin{cases} x+2y+z=8 \\ y+z=5 \\ -7y-z=-17 \end{cases} \overset{L_3 \to L_3+7L_2}{\Longrightarrow} \begin{cases} x+2y+z=8 \\ y+z=5 \\ 6z=18 \end{cases} \overset{L_3 \to 1/6\,L_3}{\Longrightarrow}$$

$$\begin{cases} x+2y+z=8 \\ y+z=5 \\ z=3 \end{cases} \overset{\substack{L_1 \to L_1-L_3 \\ L_2 \to L_2-L_3}}{\Longrightarrow} \begin{cases} x+2y=5 \\ y=2 \\ z=3 \end{cases} \overset{L_1 \to L_1-2L_2}{\Longrightarrow}$$

$$\begin{cases} x=1 \\ y=2 \\ z=3 \end{cases}$$

$S = \Big\{(1\,;2\,;3)\Big\}$

2) $$\begin{cases} 2x-y+3z=4 \\ 3x+4y-z=-5 \\ x+5y-4z=-9 \end{cases} \overset{L_1 \leftrightarrow L_3}{\Longrightarrow} \begin{cases} x+5y-4z=-9 \\ 3x+4y-z=-5 \\ 2x-y+3z=4 \end{cases} \overset{\substack{L_2 \to L_2-3L_1 \\ L_3 \to L_3-2L_1}}{\Longrightarrow}$$

$$\begin{cases} x+5y-4z=-9 \\ -11y+11z=22 \\ -11y+11z=22 \end{cases} \overset{\substack{L_2 \to -1/11\,L_2 \\ L_3 \to -1/11\,L_3}}{\Longrightarrow} \begin{cases} x+5y-4z=-9 \\ y-z=-2 \\ y-z=-2 \end{cases} \overset{L_3 \to L_3-L_2}{\Longrightarrow}$$

$$\begin{cases} x+5y-4z=-9 \\ y-z=-2 \\ 0=0 \end{cases}$$ vrai pour n'importe quelle valeur de z

$$\begin{cases} x+5y-4z=-9 \\ y-z=-2 \\ z=\alpha \end{cases} \text{ où } \alpha \in \mathbb{R} \overset{\substack{L_1 \to L_1+4L_3 \\ L_2 \to L_2+L_3}}{\Longrightarrow} \begin{cases} x+5y=-9+4\alpha \\ y=-2+\alpha \\ z=\alpha \end{cases} \text{ où } \alpha \in \mathbb{R}$$

$$\overset{L_1 \to L_1-5L_2}{\Longrightarrow} \begin{cases} x=1-\alpha \\ y=-2+\alpha \\ z=\alpha \end{cases} \text{ où } \alpha \in \mathbb{R}$$

$S = \Big\{(1-\alpha\,;-2+\alpha\,;\alpha) : \alpha \in \mathbb{R}\Big\}$

3) $$\begin{cases} 2x+y+3z=3 \\ 3x-y+4z=2 \\ 4x+y-z=5 \\ x+y+z=4 \end{cases} \overset{L_1 \leftrightarrow L_4}{\Longrightarrow} \begin{cases} x+y+z=4 \\ 3x-y+4z=2 \\ 4x+y-z=5 \\ 2x+y+3z=3 \end{cases} \overset{\substack{L_2 \to L_2-3L_1 \\ L_3 \to L_3-4L_1 \\ L_4 \to L_4-2L_1}}{\Longrightarrow}$$

$$\begin{cases} x+y+z=4 \\ -4y+z=-10 \\ -3y-5z=-11 \\ -y+z=-5 \end{cases} \overset{L_2 \leftrightarrow L_4}{\Longrightarrow} \begin{cases} x+y+z=4 \\ -y+z=-5 \\ -3y-5z=-11 \\ -4y+z=-10 \end{cases} \overset{\substack{L_3 \to L_3-3L_2 \\ L_4 \to L_4-4L_2}}{\Longrightarrow}$$

$$\begin{cases} x + y + z = 4 \\ -y + z = -5 \\ -8z = 4 \\ -3z = 10 \end{cases} \xRightarrow{L_4 \to 8L_4 - 3L_3} \begin{cases} x + y + z = 4 \\ -y + z = -5 \\ -8z = 4 \\ 0 = 68 \quad \text{impossible} \end{cases}$$

$S = \varnothing$

4) $$\begin{cases} 2y + z = -2 \\ 3x + 5y - 5z = 1 \\ 2x + 4y - 2z = 2 \end{cases} \xRightarrow{L_1 \leftrightarrow 1/3\,L_3} \begin{cases} x + 2y - z = 1 \\ 3x + 5y - 5z = 1 \\ 2y + z = -2 \end{cases} \xRightarrow{L_2 \to L_2 - 3L_1}$$

$$\begin{cases} x + 2y - z = 1 \\ -y - 2z = -2 \\ 2y + z = -2 \end{cases} \xRightarrow{L_3 \to L_3 + 2L_2} \begin{cases} x + 2y - z = 1 \\ -y - 2z = -2 \\ -3z = -6 \end{cases} \xRightarrow[L_3 \to -1/3\,L_3]{L_2 \to -L_2}$$

$$\begin{cases} x + 2y - z = 1 \\ y + 2z = 2 \\ z = 2 \end{cases} \xRightarrow[L_2 \to L_2 - 2L_3]{L_1 \to L_1 + L_3} \begin{cases} x + 2y = 3 \\ y = -2 \\ z = 2 \end{cases} \xRightarrow{L_1 \to L_1 - 2L_2}$$

$$\begin{cases} x = 7 \\ y = -2 \\ z = 2 \end{cases}$$

$S = \big\{(7\,;-2\,;2)\big\}$

5) $$\begin{cases} 2x - 3y + z = -1 \\ -6x + 9y - 3z = 3 \end{cases} \xRightarrow{L_2 \to L_2 + 3L_1} \begin{cases} 2x - 3y + z = -1 \\ 0 = 0 \end{cases}$$

On a affaire ici à deux variables libres : y et z.

$$\begin{cases} 2x - 3y + z = -1 \\ y = \alpha \\ z = \beta \end{cases} \xRightarrow{L_1 \to L_1 + 3L_2 - L_3} \begin{cases} 2x = -1 + 3\alpha - \beta \\ y = \alpha \\ z = \beta \end{cases}$$

$$\xRightarrow{L_1 \to 1/2\,L_1} \begin{cases} x = -\frac{1}{2} + \frac{3}{2}\alpha - \frac{1}{2}\beta \\ y = \alpha \\ z = \beta \end{cases}$$

$S = \Big\{(-\frac{1}{2} + \frac{3}{2}\alpha - \frac{1}{2}\beta\,;\alpha\,;\beta) : \alpha, \beta \in \mathbb{R}\Big\}$ est correct, mais peut être simplifié.

En posant $\alpha = 1$ et $\beta = 0$, on obtient la solution particulière :

$$\begin{cases} x = -\frac{1}{2} + \frac{3}{2}\cdot 1 - \frac{1}{2}\cdot 0 = 1 \\ y = 1 \\ z = 0 \end{cases}$$

On obtient $\begin{cases} x = 1 + \frac{3}{2}\alpha - \frac{1}{2}\beta \\ y = 1 + \alpha \\ z = \beta \end{cases}$ comme solution provisoire.

On simplifie encore en mettant $\frac{1}{2}$ en évidence :

$$\begin{cases} x = 1 + 3\cdot\frac{1}{2}\alpha - 1\cdot\frac{1}{2}\beta = 1 + 3\alpha - \beta \\ y = 1 + 2\cdot\frac{1}{2}\alpha = 1 + 2\alpha \\ z = 2\cdot\frac{1}{2}\beta = 2\beta \end{cases} \quad \text{où } \alpha, \beta \in \mathbb{R}$$

$S = \Big\{(1+3\,\alpha-\beta\,;1+2\,\alpha\,;2\,\beta) : \alpha,\beta\in\mathbb{R}\Big\}$

6) $\begin{cases} 2\,x + 4\,y - 3\,z = 5 \\ 5\,x + 10\,y - 7\,z = 13 \\ 3\,x + 6\,y + 5\,z = 17 \end{cases} \overset{\substack{L_2 \to 2\,L_2 - 5\,L_1 \\ L_3 \to 2\,L_2 - 3\,L_1}}{\Longrightarrow} \begin{cases} 2\,x + 4\,y - 3\,z = 5 \\ z = 1 \\ 19\,z = 19 \end{cases} \overset{L_3 \to L_3 - 19\,L_2}{\Longrightarrow}$

$\begin{cases} 2\,x + 4\,y - 3\,z = 5 \\ z = 1 \\ 19\,z = 19 \end{cases} \overset{\substack{L_1 \to L_1 + 3\,L_2 \\ L_3 \to L_3 - 19\,L_2}}{\Longrightarrow} \begin{cases} 2\,x + 4\,y = 8 \\ z = 1 \\ 0 = 0 \end{cases}$ vrai pour n'importe quelle valeur de y

$\overset{L_1 \to {}^1\!/_2\,L_1}{\Longrightarrow} \begin{cases} x + 2\,y = 4 \\ y = \alpha \\ z = 1 \end{cases} \overset{L_1 \to L_1 - 2\,L_2}{\Longrightarrow} \begin{cases} x = 4 - 2\,\alpha \\ y = \alpha \\ z = 1 \end{cases}$ où $\alpha\in\mathbb{R}$

$S = \Big\{(4-2\,\alpha\,;\alpha\,;1) : \alpha\in\mathbb{R}\Big\}$