1.5 1) $\begin{cases} x+2y-5z+4t=1 \\ 2x-3y+2z+3t=18 \\ 4x-7y+z-6t=-5 \\ x+y-z+t=1 \end{cases} \overset{\substack{L_2\to L_2-2L_1\\ L_3\to L_3-4L_1\\ L_4\to L_4-L_1}}{\Longrightarrow} \begin{cases} x+2y-5z+4t=1 \\ -7y+12z-5t=16 \\ -15y+21z-22t=-9 \\ -y+4z-3t=0 \end{cases}$

$\overset{L_2\leftrightarrow -L_4}{\Longrightarrow} \begin{cases} x+2y-5z+4t=1 \\ y-4z+3t=0 \\ -15y+21z-22t=-9 \\ -7y+12z-5t=16 \end{cases} \overset{\substack{L_3\to L_3+15L_2\\ L_4\to L_4+7L_2}}{\Longrightarrow}$

$\begin{cases} x+2y-5z+4t=1 \\ y-4z+3t=0 \\ -39z+23t=-9 \\ -16z+16t=16 \end{cases} \overset{L_3\leftrightarrow -1/16\,L_4}{\Longrightarrow} \begin{cases} x+2y-5z+4t=1 \\ y-4z+3t=0 \\ z-t=-1 \\ -39z+23t=-9 \end{cases}$

$\overset{L_4\to L_4+39L_3}{\Longrightarrow} \begin{cases} x+2y-5z+4t=1 \\ y-4z+3t=0 \\ z-t=-1 \\ -16t=-48 \end{cases} \overset{L_4\to -1/16\,L_4}{\Longrightarrow}$

$\begin{cases} x+2y-5z+4t=1 \\ y-4z+3t=0 \\ z-t=-1 \\ t=3 \end{cases} \overset{\substack{L_1\to L_1-4L_4\\ L_2\to L_2-3L_4\\ L_3\to L_3+L_4}}{\Longrightarrow} \begin{cases} x+2y-5z=-11 \\ y-4z=-9 \\ z=2 \\ t=3 \end{cases}$

$\overset{\substack{L_1\to L_1+5L_3\\ L_2\to L_2+4L_3}}{\Longrightarrow} \begin{cases} x+2y=-1 \\ y=-1 \\ z=2 \\ t=3 \end{cases} \overset{L_1\to L_1-2L_2}{\Longrightarrow} \begin{cases} x=1 \\ y=-1 \\ z=2 \\ t=3 \end{cases}$

$S=\big\{(1\,;-1\,;2\,;3)\big\}$

2) $\begin{cases} x-3y+z-t=0 \\ 2x+y-z+2t=0 \end{cases} \overset{L_2\to L_2-2L_1}{\Longrightarrow} \begin{cases} x-3y+z-t=0 \\ 7y-3z+4t=0 \end{cases}$

$\overset{L_1\to 7L_1+3L_2}{\Longrightarrow} \begin{cases} 7x-2z+5t=0 \\ 7y-3z+4t=0 \end{cases} \overset{\substack{L_1\to 1/7\,L_1\\ L_2\to 1/7\,L_2}}{\Longrightarrow} \begin{cases} x-\frac{2}{7}z+\frac{5}{7}t=0 \\ y-\frac{3}{7}z+\frac{4}{7}t=0 \end{cases}$

Vu que les variables z et t sont libres, on a :

$\begin{cases} x=\frac{2}{7}\alpha-\frac{5}{7}\beta=2\cdot\frac{1}{7}\alpha-5\cdot\frac{1}{7}\beta \\ y=\frac{3}{7}\alpha-\frac{4}{7}\beta=3\cdot\frac{1}{7}\alpha-4\cdot\frac{1}{7}\beta \\ z=\alpha=7\cdot\frac{1}{7}\alpha \\ t=\beta=7\cdot\frac{1}{7}\beta \end{cases}$ où $\alpha,\beta\in\mathbb{R}$

$S=\big\{(2\,\alpha-5\,\beta\,;3\,\alpha-4\,\beta\,;7\,\alpha\,;7\,\beta):\alpha,\beta\in\mathbb{R}\big\}.$

3) $\begin{cases} 2x + y - 3z + t + u = 4 \\ x - 2y - z + 3t - u = 1 \\ 3x - y + 4z - t - 3u = -6 \\ x + y + z + t + u = 15 \\ 5x - 4y + 3z - 2t + u = 3 \end{cases} \overset{L_1 \leftrightarrow L_2}{\Longrightarrow} \begin{cases} x - 2y - z + 3t - u = 1 \\ 2x + y - 3z + t + u = 4 \\ 3x - y + 4z - t - 3u = -6 \\ x + y + z + t + u = 15 \\ 5x - 4y + 3z - 2t + u = 3 \end{cases}$

$\begin{matrix} L_2 \to L_2 - 2L_1 \\ L_3 \to L_3 - 3L_1 \\ L_4 \to L_4 - L_1 \\ L_5 \to L_5 - 5L_1 \end{matrix} \Longrightarrow \begin{cases} x - 2y - z + 3t - u = 1 \\ 5y - z - 5t + 3u = 2 \\ 5y + 7z - 10t = -9 \\ 3y + 2z - 2t + 2u = 14 \\ 6y + 8z - 17t + 6u = -2 \end{cases} \begin{matrix} L_3 \to L_3 - L_2 \\ L_4 \to 5L_4 - 3L_2 \\ L_5 \to 5L_5 - 6L_2 \\ \Longrightarrow \end{matrix}$

$\begin{cases} x - 2y - z + 3t - u = 1 \\ 5y - z - 5t + 3u = 2 \\ 8z - 5t - 3u = -11 \\ 13z + 5t + u = 64 \\ 46z - 55t + 12u = -22 \end{cases} \begin{matrix} L_4 \to 8L_4 - 13L_3 \\ L_5 \to 4L_5 - 23L_3 \\ \Longrightarrow \end{matrix}$

$\begin{cases} x - 2y - z + 3t - u = 1 \\ 5y - z - 5t + 3u = 2 \\ 8z - 5t - 3u = -11 \\ 105t + 47u = 655 \\ -105t + 117u = 165 \end{cases} \overset{L_5 \to L_5 + L_4}{\Longrightarrow}$

$\begin{cases} x - 2y - z + 3t - u = 1 \\ 5y - z - 5t + 3u = 2 \\ 8z - 5t - 3u = -11 \\ 105t + 47u = 655 \\ 164u = 820 \end{cases} \overset{L_5 \to 1/164 L_5}{\Longrightarrow}$

$\begin{cases} x - 2y - z + 3t - u = 1 \\ 5y - z - 5t + 3u = 2 \\ 8z - 5t - 3u = -11 \\ 105t + 47u = 655 \\ u = 5 \end{cases} \begin{matrix} L_1 \to L_1 + L_5 \\ L_2 \to L_2 - 3L_5 \\ L_3 \to L_3 + 3L_5 \\ L_4 \to L_4 - 47L_5 \\ \Longrightarrow \end{matrix}$

$\begin{cases} x - 2y - z + 3t = 6 \\ 5y - z - 5t = -13 \\ 8z - 5t = 4 \\ 105t = 420 \\ u = 5 \end{cases} \overset{L_4 \to 1/105 L_4}{\Longrightarrow} \begin{cases} x - 2y - z + 3t = 6 \\ 5y - z - 5t = -13 \\ 8z - 5t = 4 \\ t = 4 \\ u = 5 \end{cases}$

$\begin{matrix} L_1 \to L_1 - 3L_4 \\ L_2 \to L_2 + 5L_4 \\ L_3 \to L_3 + 5L_4 \\ \Longrightarrow \end{matrix} \begin{cases} x - 2y - z = -6 \\ 5y - z = 7 \\ 8z = 24 \\ t = 4 \\ u = 5 \end{cases} \overset{L_3 \to 1/8 L_3}{\Longrightarrow} \begin{cases} x - 2y - z = -6 \\ 5y - z = 7 \\ z = 3 \\ t = 4 \\ u = 5 \end{cases}$

$$\begin{array}{c} \scriptstyle \mathrm{L}_1 \to \mathrm{L}_1 + \mathrm{L}_3 \\ \scriptstyle \mathrm{L}_2 \to \mathrm{L}_2 + \mathrm{L}_3 \\ \Longrightarrow \end{array} \left\{ \begin{array}{rrrrrrrrr} x & - & 2\,y & & & & & = & -3 \\ & & 5\,y & & & & & = & 10 \\ & & & & z & & & = & 3 \\ & & & & & & t & = & 4 \\ & & & & & & & u = & 5 \end{array} \right. \begin{array}{c} \scriptstyle \mathrm{L}_2 \to 1/5\,\mathrm{L}_2 \\ \Longrightarrow \end{array} \left\{ \begin{array}{rrrrrrrrr} x & - & 2\,y & & & & & = & -3 \\ & & y & & & & & = & 2 \\ & & & & z & & & = & 3 \\ & & & & & & t & = & 4 \\ & & & & & & & u = & 5 \end{array} \right.$$

$$\begin{array}{c} \scriptstyle \mathrm{L}_1 \to \mathrm{L}_1 + 2\,\mathrm{L}_2 \\ \Longrightarrow \end{array} \left\{ \begin{array}{rrrrrrrrr} x & & & & & & & = & 1 \\ & & y & & & & & = & 2 \\ & & & & z & & & = & 3 \\ & & & & & & t & = & 4 \\ & & & & & & & u = & 5 \end{array} \right.$$

$\mathrm{S} = \{(1\,;2\,;3\,;4\,;5)\}$