

1.6 1)
$$\begin{cases} x - y - z = 6 \\ x - 2y - 3z = 10 \\ 5x + 6y + z = 2 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 1 & -2 & -3 & 10 \\ 5 & 6 & 1 & 2 \end{array} \right) \xRightarrow[\text{L}_3 \rightarrow \text{L}_3 - 5\text{L}_1]{\text{L}_2 \rightarrow \text{L}_2 - \text{L}_1} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & -1 & -2 & 4 \\ 0 & 11 & 6 & -28 \end{array} \right) \xRightarrow{\text{L}_3 \rightarrow \text{L}_3 + 11\text{L}_2}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & -1 & -2 & 4 \\ 0 & 0 & -16 & 16 \end{array} \right) \xRightarrow{\text{L}_3 \rightarrow -1/16\text{L}_3} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & -1 & -2 & 4 \\ 0 & 0 & 1 & -1 \end{array} \right) \xRightarrow[\text{L}_2 \rightarrow \text{L}_2 + 2\text{L}_3]{\text{L}_1 \rightarrow \text{L}_1 + \text{L}_3}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right) \xRightarrow{\text{L}_2 \rightarrow -\text{L}_2} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array} \right) \xRightarrow{\text{L}_1 \rightarrow \text{L}_1 + \text{L}_2}$$

$$\left(\begin{array}{ccc|c} \textcircled{1} & 0 & 0 & 3 \\ 0 & \textcircled{1} & 0 & -2 \\ 0 & 0 & \textcircled{1} & -1 \end{array} \right) \Rightarrow \begin{cases} x = 3 \\ y = -2 \\ z = -1 \end{cases}$$

$S = \{(3; -2; -1)\}$

2)
$$\begin{cases} x + y + z = 9 \\ x + 2y + 3z = 14 \\ 3x + 2y + z = 22 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & 2 & 3 & 14 \\ 3 & 2 & 1 & 22 \end{array} \right) \xRightarrow[\text{L}_3 \rightarrow \text{L}_3 - 3\text{L}_1]{\text{L}_2 \rightarrow \text{L}_2 - \text{L}_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -2 & -5 \end{array} \right) \xRightarrow{\text{L}_3 \rightarrow \text{L}_3 + \text{L}_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right) \xRightarrow{\text{L}_1 \rightarrow \text{L}_1 - \text{L}_2} \left(\begin{array}{ccc|c} \textcircled{1} & 0 & -1 & 4 \\ 0 & \textcircled{1} & 2 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Il n'y a pas de pivot dans la troisième colonne : la variable z est libre :

$$\left(\begin{array}{ccc|c} \textcircled{1} & 0 & -1 & 4 \\ 0 & \textcircled{1} & 2 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right) \iff \begin{cases} x - \alpha = 4 \\ y + 2\alpha = 5 \\ z = \alpha \end{cases} \text{ où } \alpha \in \mathbb{R}$$

$$S = \{(4 + \alpha; 5 - 2\alpha; \alpha) : \alpha \in \mathbb{R}\}$$

3)
$$\begin{cases} 2x_1 + 3x_2 - x_3 + 4x_4 = 1 \\ 3x_1 - x_2 + x_4 = 1 \\ 3x_1 - 4x_2 + x_3 - x_4 = 2 \end{cases}$$

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & 4 & 1 \\ 3 & -1 & 0 & 1 & 1 \\ 3 & -4 & 1 & -1 & 2 \end{array} \right) \xRightarrow[\text{L}_3 \rightarrow 2\text{L}_3 - 3\text{L}_1]{\text{L}_2 \rightarrow 2\text{L}_2 - 3\text{L}_1} \left(\begin{array}{cccc|c} 2 & 3 & -1 & 4 & 1 \\ 0 & -11 & 3 & -10 & -1 \\ 0 & -17 & 5 & -14 & 1 \end{array} \right)$$

$$\xRightarrow{\text{L}_3 \rightarrow 11\text{L}_3 - 17\text{L}_2} \left(\begin{array}{cccc|c} 2 & 3 & -1 & 4 & 1 \\ 0 & -11 & 3 & -10 & -1 \\ 0 & 0 & 4 & 16 & 28 \end{array} \right) \xRightarrow{\text{L}_3 \rightarrow 1/4\text{L}_3}$$

$$\begin{aligned}
& \left(\begin{array}{cccc|c} 2 & 3 & -1 & 4 & 1 \\ 0 & -11 & 3 & -10 & -1 \\ 0 & 0 & 1 & 4 & 7 \end{array} \right) \xrightarrow[\text{L}_2 \rightarrow \text{L}_2 - 3\text{L}_3]{\text{L}_1 \rightarrow \text{L}_1 + \text{L}_3} \left(\begin{array}{cccc|c} 2 & 3 & 0 & 8 & 8 \\ 0 & -11 & 0 & -22 & -22 \\ 0 & 0 & 1 & 4 & 7 \end{array} \right) \\
& \xrightarrow{\text{L}_2 \rightarrow -1/11 \text{L}_2} \left(\begin{array}{cccc|c} 2 & 3 & 0 & 8 & 8 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 4 & 7 \end{array} \right) \xrightarrow{\text{L}_1 \rightarrow \text{L}_1 - 3\text{L}_2} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 4 & 7 \end{array} \right) \\
& \xrightarrow{\text{L}_1 \rightarrow 1/2 \text{L}_1} \left(\begin{array}{cccc|c} \textcircled{1} & 0 & 0 & 1 & 1 \\ 0 & \textcircled{1} & 0 & 2 & 2 \\ 0 & 0 & \textcircled{1} & 4 & 7 \end{array} \right)
\end{aligned}$$

Il n'y a pas de pivot dans la quatrième colonne : la variable x_4 est libre.

$$\left(\begin{array}{cccc|c} \textcircled{1} & 0 & 0 & 1 & 1 \\ 0 & \textcircled{1} & 0 & 2 & 2 \\ 0 & 0 & \textcircled{1} & 4 & 7 \end{array} \right) \iff \begin{cases} x_1 + \alpha = 1 \\ x_2 + 2\alpha = 2 \\ x_3 + 4\alpha = 7 \\ x_4 = \alpha \end{cases}$$

$$S = \{(1 - \alpha; 2 - 2\alpha; 7 - 4\alpha; \alpha) : \alpha \in \mathbb{R}\}$$

$$4) \begin{cases} x_1 + x_2 + 2x_3 + x_4 = 1 \\ x_1 - x_2 - x_3 + x_4 = 0 \\ x_2 + x_3 = -1 \\ x_1 + x_2 + x_4 = 2 \end{cases}$$

$$\begin{aligned}
& \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 1 & 1 & 0 & 1 & 2 \end{array} \right) \xrightarrow[\text{L}_4 \rightarrow \text{L}_4 - \text{L}_1]{\text{L}_2 \rightarrow \text{L}_2 - \text{L}_1} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & -2 & -3 & 0 & -1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & -2 & 0 & 1 \end{array} \right) \xrightarrow{\text{L}_2 \leftrightarrow \text{L}_3} \\
& \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -2 & -3 & 0 & -1 \\ 0 & 0 & -2 & 0 & 1 \end{array} \right) \xrightarrow{\text{L}_3 \rightarrow \text{L}_3 + 2\text{L}_2} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & -2 & 0 & 1 \end{array} \right) \xrightarrow{\text{L}_4 \rightarrow \text{L}_4 - 2\text{L}_3} \\
& \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 7 \end{array} \right)
\end{aligned}$$

La dernière équation équivaut à $0 = 7$ qui est impossible.

$$S = \emptyset$$

$$5) \begin{cases} x_1 + x_2 - x_3 + x_4 - x_5 = 8 \\ -2x_1 + x_2 - 2x_3 + 3x_4 = 6 \\ 3x_1 - 2x_2 - x_3 + x_4 + 2x_5 = 8 \\ x_1 + 3x_2 - 2x_4 - 5x_5 = 1 \\ x_1 + 2x_2 + 3x_3 + x_4 - 3x_5 = -1 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ -2 & 1 & -2 & 3 & 0 & 6 \\ 3 & -2 & -1 & 1 & 2 & 8 \\ 1 & 3 & 0 & -2 & -5 & 1 \\ 1 & 2 & 3 & 1 & -3 & -1 \end{array} \right) \xRightarrow{\begin{array}{l} L_2 \rightarrow L_2 + 2L_1 \\ L_3 \rightarrow L_3 - 3L_1 \\ L_4 \rightarrow L_4 - L_1 \\ L_5 \rightarrow L_5 - L_1 \end{array}} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 3 & -4 & 5 & -2 & 22 \\ 0 & -5 & 2 & -2 & 5 & -16 \\ 0 & 2 & 1 & -3 & -4 & -7 \\ 0 & 1 & 4 & 0 & -2 & -9 \end{array} \right)$$

$$\xRightarrow{L_2 \leftrightarrow L_5} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & -5 & 2 & -2 & 5 & -16 \\ 0 & 2 & 1 & -3 & -4 & -7 \\ 0 & 3 & -4 & 5 & -2 & 22 \end{array} \right) \xRightarrow{\begin{array}{l} L_3 \rightarrow L_3 + 5L_2 \\ L_4 \rightarrow L_4 - 2L_2 \\ L_5 \rightarrow L_5 - 3L_2 \end{array}}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & 0 & 22 & -2 & -5 & -61 \\ 0 & 0 & -7 & -3 & 0 & 11 \\ 0 & 0 & -16 & 5 & 4 & 49 \end{array} \right) \xRightarrow{L_4 \leftrightarrow -L_3} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 22 & -2 & -5 & -61 \\ 0 & 0 & -16 & 5 & 4 & 49 \end{array} \right)$$

$$\xRightarrow{\begin{array}{l} L_4 \rightarrow 7L_4 - 22L_3 \\ L_5 \rightarrow 7L_5 + 16L_3 \end{array}} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 0 & -80 & -35 & -185 \\ 0 & 0 & 0 & 83 & 28 & 167 \end{array} \right) \xRightarrow{L_4 \rightarrow -\frac{1}{5}L_4}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 0 & 16 & 7 & 37 \\ 0 & 0 & 0 & 83 & 28 & 167 \end{array} \right) \xRightarrow{L_5 \rightarrow 16L_5 - 83L_4} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 0 & 16 & 7 & 37 \\ 0 & 0 & 0 & 0 & -133 & -399 \end{array} \right)$$

$$\xRightarrow{L_5 \rightarrow -1/133L_5} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -1 & 8 \\ 0 & 1 & 4 & 0 & -2 & -9 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 0 & 16 & 7 & 37 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right) \xRightarrow{\begin{array}{l} L_1 \rightarrow L_1 + L_5 \\ L_2 \rightarrow L_2 + 2L_5 \\ L_4 \rightarrow L_4 - 7L_5 \end{array}}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & 0 & 11 \\ 0 & 1 & 4 & 0 & 0 & -3 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 0 & 16 & 0 & 16 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right) \xRightarrow{L_4 \rightarrow 1/16L_4} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & 0 & 11 \\ 0 & 1 & 4 & 0 & 0 & -3 \\ 0 & 0 & 7 & 3 & 0 & -11 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right) \xRightarrow{\begin{array}{l} L_1 \rightarrow L_1 - L_4 \\ L_3 \rightarrow L_3 - 3L_4 \end{array}}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 0 & 0 & 10 \\ 0 & 1 & 4 & 0 & 0 & -3 \\ 0 & 0 & 7 & 0 & 0 & -14 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right) \xRightarrow{L_3 \rightarrow 1/7L_3} \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 0 & 0 & 10 \\ 0 & 1 & 4 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right) \xRightarrow{\begin{array}{l} L_1 \rightarrow L_1 + L_3 \\ L_2 \rightarrow L_2 - 4L_3 \end{array}}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right) \xrightarrow{L_1 \rightarrow L_1 - L_2} \left(\begin{array}{ccccc|c} \textcircled{1} & 0 & 0 & 0 & 0 & 3 \\ 0 & \textcircled{1} & 0 & 0 & 0 & 5 \\ 0 & 0 & \textcircled{1} & 0 & 0 & -2 \\ 0 & 0 & 0 & \textcircled{1} & 0 & 1 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 3 \end{array} \right)$$

$$S = \{(3; 5; -2; 1; 3)\}$$

$$6) \begin{cases} 3x_1 - 3x_2 - x_3 + 2x_4 - 9x_5 = 13 \\ x_1 - x_2 + 2x_3 - x_4 - 6x_5 = -6 \\ x_1 - x_2 + x_3 + x_4 - 6x_5 = 1 \\ -x_1 + x_2 - x_3 - 2x_4 + 7x_5 = -3 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 3 & -3 & -1 & 2 & -9 & 13 \\ 1 & -1 & 2 & -1 & -6 & -6 \\ 1 & -1 & 1 & 1 & -6 & 1 \\ -1 & 1 & -1 & -2 & 7 & -3 \end{array} \right) \xrightarrow{L_1 \leftrightarrow L_2} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & -1 & -6 & -6 \\ 3 & -3 & -1 & 2 & -9 & 13 \\ 1 & -1 & 1 & 1 & -6 & 1 \\ -1 & 1 & -1 & -2 & 7 & -3 \end{array} \right)$$

$$\begin{array}{l} L_2 \rightarrow L_2 - 3L_1 \\ L_3 \rightarrow L_3 - L_1 \\ L_4 \rightarrow L_4 + L_1 \end{array} \xRightarrow{} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & -1 & -6 & -6 \\ 0 & 0 & -7 & 5 & 9 & 31 \\ 0 & 0 & -1 & 2 & 0 & 7 \\ 0 & 0 & 1 & -3 & 1 & -9 \end{array} \right) \xrightarrow{L_2 \leftrightarrow L_4}$$

$$\left(\begin{array}{ccccc|c} 1 & -1 & 2 & -1 & -6 & -6 \\ 0 & 0 & 1 & -3 & 1 & -9 \\ 0 & 0 & -1 & 2 & 0 & 7 \\ 0 & 0 & -7 & 5 & 9 & 31 \end{array} \right) \xrightarrow{\begin{array}{l} L_3 \rightarrow L_3 + L_2 \\ L_4 \rightarrow L_4 + 7L_2 \end{array}} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & -1 & -6 & -6 \\ 0 & 0 & 1 & -3 & 1 & -9 \\ 0 & 0 & 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & -16 & 16 & -32 \end{array} \right)$$

$$\begin{array}{l} L_3 \rightarrow -L_3 \\ L_4 \rightarrow -1/16 L_4 \end{array} \xRightarrow{} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & -1 & -6 & -6 \\ 0 & 0 & 1 & -3 & 1 & -9 \\ 0 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 & 2 \end{array} \right) \xrightarrow{L_4 \rightarrow L_4 - L_3}$$

$$\left(\begin{array}{ccccc|c} 1 & -1 & 2 & -1 & -6 & -6 \\ 0 & 0 & 1 & -3 & 1 & -9 \\ 0 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\begin{array}{l} L_1 \rightarrow L_1 + L_3 \\ L_2 \rightarrow L_2 + 3L_3 \end{array}} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & -7 & -4 \\ 0 & 0 & 1 & 0 & -2 & -3 \\ 0 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$L_1 \rightarrow L_1 - 2L_2 \xRightarrow{} \left(\begin{array}{ccccc|c} \textcircled{1} & -1 & 0 & 0 & -3 & 2 \\ 0 & 0 & \textcircled{1} & 0 & -2 & -3 \\ 0 & 0 & 0 & \textcircled{1} & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Pas de pivot dans les colonnes 2 et 5 : les variables x_2 et x_5 sont libres.

$$\left(\begin{array}{ccccc|c} \textcircled{1} & -1 & 0 & 0 & -3 & 2 \\ 0 & 0 & \textcircled{1} & 0 & -2 & -3 \\ 0 & 0 & 0 & \textcircled{1} & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \iff \begin{cases} x_1 - \alpha - 3\beta = 2 \\ x_2 = \alpha \\ x_3 - 2\beta = -3 \\ x_4 - \beta = 2 \\ x_5 = \beta \end{cases}$$

$$S = \{(2 + \alpha + 3\beta; \alpha; -3 + 2\beta; 2 + \beta; \beta) : \alpha, \beta \in \mathbb{R}\}$$

$$7) \begin{cases} x_1 - x_2 + 2x_3 + 3x_5 = 1 \\ -x_1 + x_2 + 2x_4 - 5x_5 = 5 \\ x_1 - x_2 + 4x_3 + 2x_4 + 4x_5 = 13 \\ -2x_1 + 2x_2 - 5x_3 - x_4 - 3x_5 = -1 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ -1 & 1 & 0 & 2 & -5 & 5 \\ 1 & -1 & 4 & 2 & 4 & 13 \\ -2 & 2 & -5 & -1 & -3 & -1 \end{array} \right) \xRightarrow{\substack{L_2 \rightarrow L_2 + L_1 \\ L_3 \rightarrow L_3 - L_1 \\ L_4 \rightarrow L_4 + 2L_1}} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 2 & 2 & -2 & 6 \\ 0 & 0 & 2 & 2 & 1 & 12 \\ 0 & 0 & -1 & -1 & 3 & 1 \end{array} \right)$$

$$\xRightarrow{L_2 \rightarrow 1/2 L_2} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & -1 & 3 \\ 0 & 0 & 2 & 2 & 1 & 12 \\ 0 & 0 & -1 & -1 & 3 & 1 \end{array} \right) \xRightarrow{\substack{L_3 \rightarrow L_3 - 2L_2 \\ L_4 \rightarrow L_4 + L_2}}$$

$$\left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{array} \right) \xRightarrow{\substack{L_3 \rightarrow 1/3 L_3 \\ L_4 \rightarrow 1/2 L_4}} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array} \right) \xRightarrow{L_4 \rightarrow L_4 - L_3}$$

$$\left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xRightarrow{\substack{L_1 \rightarrow L_1 - 3L_3 \\ L_2 \rightarrow L_2 + L_3}} \left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 0 & -5 \\ 0 & 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xRightarrow{L_1 \rightarrow L_1 - 2L_2}$$

$$\left(\begin{array}{ccccc|c} \textcircled{1} & -1 & 0 & -2 & 0 & -15 \\ 0 & 0 & \textcircled{1} & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Pas de pivot dans les colonnes 2 et 4 : les variables x_2 et x_4 sont libres.

$$\left(\begin{array}{ccccc|c} \textcircled{1} & -1 & 0 & -2 & 0 & -15 \\ 0 & 0 & \textcircled{1} & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \iff \begin{cases} x_1 - \alpha - 2\beta = -15 \\ x_2 = \alpha \\ x_3 + \beta = 5 \\ x_4 = \beta \\ x_5 = 2 \end{cases}$$

$$S = \{(-15 + \alpha + 2\beta; \alpha; 5 - \beta; \beta; 2) : \alpha, \beta \in \mathbb{R}\}$$