2.2 1) $\alpha\,\boldsymbol{v_1} + \beta\,\boldsymbol{v_2} + \gamma\,\boldsymbol{v_3} = \boldsymbol{v} \iff \alpha\begin{pmatrix}1\\1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\\1\end{pmatrix} + \gamma\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} \iff$

$$\begin{cases}\alpha \quad\quad + \gamma = 1\\ \alpha + \beta \quad\quad = 2\\ \quad\quad \beta + \gamma = 3\end{cases}$$

$$\left(\begin{array}{ccc|c}1&0&1&1\\1&1&0&2\\0&1&1&3\end{array}\right) \overset{L_2 \to L_2 - L_1}{\Longrightarrow} \left(\begin{array}{ccc|c}1&0&1&1\\0&1&-1&1\\0&1&1&3\end{array}\right) \overset{L_3 \to L_3 - L_2}{\Longrightarrow} \left(\begin{array}{ccc|c}1&0&1&1\\0&1&-1&1\\0&0&2&2\end{array}\right)$$

$$\overset{L_3 \to 1/2\,L_3}{\Longrightarrow} \left(\begin{array}{ccc|c}1&0&1&1\\0&1&-1&1\\0&0&1&1\end{array}\right) \overset{\substack{L_1 \to L_1 - L_3\\ L_2 \to L_2 + L_3}}{\Longrightarrow} \left(\begin{array}{ccc|c}1&0&0&0\\0&1&0&2\\0&0&1&1\end{array}\right)$$

$\boldsymbol{v} = 2\,\boldsymbol{v_2} + \boldsymbol{v_3} = 0\,\boldsymbol{v_1} + 2\,\boldsymbol{v_2} + \boldsymbol{v_3}$ est bien une combinaison linéaire des vecteurs $\boldsymbol{v_1}$, $\boldsymbol{v_2}$ et $\boldsymbol{v_3}$.

2) $\alpha\,\boldsymbol{v_1} + \beta\,\boldsymbol{v_2} + \gamma\,\boldsymbol{v_3} = \boldsymbol{v} \iff \alpha\begin{pmatrix}10\\4\\48\end{pmatrix} + \beta\begin{pmatrix}34\\14\\-64\end{pmatrix} + \gamma\begin{pmatrix}-12\\2\\-10\end{pmatrix} = \begin{pmatrix}32\\20\\-26\end{pmatrix} \iff$

$$\begin{cases}10\,\alpha + 34\,\beta - 12\,\gamma = 32\\ 4\,\alpha + 14\,\beta + 2\,\gamma = 20\\ 48\,\alpha - 64\,\beta - 10\,\gamma = -26\end{cases}$$

$$\left(\begin{array}{ccc|c}10&34&-12&32\\4&14&2&20\\48&-64&-10&-26\end{array}\right) \overset{\substack{L_2 \to 5\,L_2 - 2\,L_1\\ L_3 \to 5\,L_3 - 24\,L_1}}{\Longrightarrow} \left(\begin{array}{ccc|c}10&34&-12&32\\0&2&34&36\\0&-1136&238&-898\end{array}\right)$$

$$\overset{L_3 \to L_3 + 568\,L_2}{\Longrightarrow} \left(\begin{array}{ccc|c}10&34&-12&32\\0&2&34&36\\0&0&19\,550&19\,550\end{array}\right) \overset{\substack{L_1 \to 1/2\,L_1 L_2 \to 1/2\,L_2\\ L_3 \to 1/19550\,L_3}}{\Longrightarrow}$$

$$\left(\begin{array}{ccc|c}5&17&-6&16\\0&1&17&18\\0&0&1&1\end{array}\right) \overset{\substack{L_1 \to L_1 + 6\,L_3\\ L_2 \to L_2 - 17\,L_3}}{\Longrightarrow} \left(\begin{array}{ccc|c}5&17&0&22\\0&1&0&1\\0&0&1&1\end{array}\right) \overset{L_1 \to L_1 - 17\,L_2}{\Longrightarrow}$$

$$\left(\begin{array}{ccc|c}5&0&0&5\\0&1&0&1\\0&0&1&1\end{array}\right) \overset{L_1 \to 1/5\,L_1}{\Longrightarrow} \left(\begin{array}{ccc|c}1&0&0&1\\0&1&0&1\\0&0&1&1\end{array}\right)$$

$\boldsymbol{v} = \boldsymbol{v_1} + \boldsymbol{v_2} + \boldsymbol{v_3}$ est une combinaison linéaire des vecteurs $\boldsymbol{v_1}$, $\boldsymbol{v_2}$ et $\boldsymbol{v_3}$.