

5.9

$$1) \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & 4 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \rightarrow L_2 + L_1} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 6 & 1 & 1 \end{array} \right) \xRightarrow{L_1 \rightarrow 3L_1 - L_2} \left(\begin{array}{cc|cc} 3 & 0 & 2 & -1 \\ 0 & 6 & 1 & 1 \end{array} \right) \xRightarrow{L_1 \rightarrow \frac{1}{3}L_1, L_2 \rightarrow \frac{1}{6}L_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & \frac{1}{6} & \frac{1}{6} \end{array} \right)$$

$$\left(\begin{array}{cc} 1 & 2 \\ -1 & 4 \end{array} \right)^{-1} = \left(\begin{array}{cc} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{array} \right)$$

$$2) \left(\begin{array}{cc|cc} 2 & -5 & 1 & 0 \\ -6 & 1 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \rightarrow L_2 + 3L_1} \left(\begin{array}{cc|cc} 2 & -5 & 1 & 0 \\ 0 & -14 & 3 & 1 \end{array} \right) \xRightarrow{L_1 \rightarrow 14L_1 - 5L_2} \left(\begin{array}{cc|cc} 28 & 0 & -1 & -5 \\ 0 & -14 & 3 & 1 \end{array} \right) \xRightarrow{L_1 \rightarrow \frac{1}{28}L_1, L_2 \rightarrow -\frac{1}{14}L_2} \left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{28} & -\frac{5}{28} \\ 0 & 1 & -\frac{3}{14} & -\frac{1}{14} \end{array} \right)$$

$$\left(\begin{array}{cc} 2 & -5 \\ -6 & 1 \end{array} \right)^{-1} = \left(\begin{array}{cc} -\frac{1}{28} & -\frac{5}{28} \\ -\frac{3}{14} & -\frac{1}{14} \end{array} \right)$$

$$3) \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 3 & 6 & 0 & 0 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \rightarrow L_2 - 2L_1, L_3 \rightarrow L_3 - 3L_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 3 & -3 & -2 & 1 & 0 \\ 0 & 6 & -6 & -3 & 0 & 1 \end{array} \right) \xRightarrow{L_3 \rightarrow L_3 - 2L_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 3 & -3 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array} \right)$$

La matrice $\begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ 3 & 6 & 0 \end{pmatrix}$ n'est pas inversible, car elle est de rang < 3 , puisqu'elle a moins de 3 pivots.

$$4) \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \rightarrow L_2 - L_1, L_3 \rightarrow -L_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \right) \xRightarrow{L_1 \rightarrow L_1 - L_3, L_2 \rightarrow L_2 + 2L_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \right) \xRightarrow{L_1 \rightarrow L_1 + L_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \right)$$

$$\left(\begin{array}{ccc} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{array} \right)^{-1} = \left(\begin{array}{ccc} 0 & 1 & -1 \\ -1 & 1 & -2 \\ 0 & 0 & -1 \end{array} \right)$$

$$5) \left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \leftrightarrow -L_1} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & -1 & 0 \\ 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \rightarrow L_2 - 2L_1} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & -1 & 0 \\ 0 & 3 & -2 & 1 & 2 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array} \right) \xRightarrow{L_2 \leftrightarrow -L_3} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 & -1 \\ 0 & 3 & -2 & 1 & 2 & 0 \end{array} \right) \xRightarrow{L_3 \rightarrow L_3 - 3L_2} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & 4 & 1 & 2 & 3 \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & 4 & 1 & 2 & 3 \end{array} \right) \xRightarrow{\substack{L_1 \rightarrow 4L_1 - L_3 \\ L_2 \rightarrow 2L_2 + L_3}} \left(\begin{array}{ccc|ccc} 4 & -8 & 0 & -1 & -6 & -3 \\ 0 & 2 & 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 1 & 2 & 3 \end{array} \right)$$

$$\xRightarrow{L_1 \rightarrow L_1 + 4L_2} \left(\begin{array}{ccc|ccc} 4 & 0 & 0 & 3 & 2 & 1 \\ 0 & 2 & 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 1 & 2 & 3 \end{array} \right) \xRightarrow{\substack{L_1 \rightarrow 1/4 L_1 \\ L_2 \rightarrow 1/2 L_2 \\ L_3 \rightarrow 1/4 L_3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & 1/2 & 1/4 \\ 0 & 1 & 0 & 1/2 & 1 & 1/2 \\ 0 & 0 & 1 & 1/4 & 1/2 & 3/4 \end{array} \right)$$

$$\left(\begin{array}{ccc} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{array} \right)^{-1} = \left(\begin{array}{ccc} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{array} \right)$$

$$6) \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & -2 & 0 & 0 & 1 \end{array} \right) \xRightarrow{\substack{L_2 \rightarrow L_2 + L_1 \\ L_3 \rightarrow L_3 - 2L_1}} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 4 & 5 & 1 & 1 & 0 \\ 0 & -5 & -10 & -2 & 0 & 1 \end{array} \right) \xRightarrow{L_3 \rightarrow 4L_3 + 5L_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 4 & 5 & 1 & 1 & 0 \\ 0 & 0 & -15 & -3 & 5 & 4 \end{array} \right) \xRightarrow{\substack{L_1 \rightarrow 15L_1 + 4L_3 \\ L_2 \rightarrow 3L_2 + L_3}} \left(\begin{array}{ccc|ccc} 15 & 45 & 0 & 3 & 20 & 16 \\ 0 & 12 & 0 & 0 & 8 & 4 \\ 0 & 0 & -15 & -3 & 5 & 4 \end{array} \right)$$

$$\xRightarrow{L_1 \rightarrow 4L_1 - 15L_2} \left(\begin{array}{ccc|ccc} 60 & 0 & 0 & 12 & -40 & 4 \\ 0 & 12 & 0 & 0 & 8 & 4 \\ 0 & 0 & -15 & -3 & 5 & 4 \end{array} \right) \xRightarrow{\substack{L_1 \rightarrow 1/60 L_1 \\ L_2 \rightarrow 1/12 L_2 \\ L_3 \rightarrow -1/15 L_3}}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & -2/3 & 1/15 \\ 0 & 1 & 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 1 & 1/5 & -1/3 & -4/15 \end{array} \right) \quad \left(\begin{array}{ccc} 1 & 3 & 4 \\ -1 & 1 & 1 \\ 2 & 1 & -2 \end{array} \right)^{-1} = \left(\begin{array}{ccc} 1/5 & -2/3 & 1/15 \\ 0 & 2/3 & 1/3 \\ 1/5 & -1/3 & -4/15 \end{array} \right)$$

$$7) \left(\begin{array}{cccc|cccc} -2 & 5 & 3 & 1 & 1 & 0 & 0 & 0 \\ -9 & 2 & -5 & 6 & 0 & 1 & 0 & 0 \\ 2 & 4 & 8 & 16 & 0 & 0 & 1 & 0 \\ 4 & 8 & 16 & 32 & 0 & 0 & 0 & 1 \end{array} \right) \xRightarrow{\substack{L_2 \rightarrow 2L_2 - 9L_1 \\ L_3 \rightarrow L_3 + L_1 \\ L_4 \rightarrow L_4 + 2L_1}}$$

$$\left(\begin{array}{cccc|cccc} -2 & 5 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & -41 & -37 & 3 & -9 & 2 & 0 & 0 \\ 0 & 9 & 11 & 17 & 1 & 0 & 1 & 0 \\ 0 & 18 & 22 & 34 & 2 & 0 & 0 & 1 \end{array} \right) \xRightarrow{L_3 \leftrightarrow L_2}$$

$$\left(\begin{array}{cccc|cccc} -2 & 5 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 9 & 11 & 17 & 1 & 0 & 1 & 0 \\ 0 & -41 & -37 & 3 & -9 & 2 & 0 & 0 \\ 0 & 18 & 22 & 34 & 2 & 0 & 0 & 1 \end{array} \right) \xRightarrow{\substack{L_3 \rightarrow 9L_3 + 41L_2 \\ L_4 \rightarrow L_4 - 2L_2}}$$

$$\left(\begin{array}{cccc|cccc} -2 & 5 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 9 & 11 & 17 & 1 & 0 & 1 & 0 \\ 0 & 0 & 118 & 724 & -40 & 18 & 41 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 \end{array} \right)$$

La matrice $\begin{pmatrix} -2 & 5 & 3 & 1 \\ -9 & 2 & -5 & 6 \\ 2 & 4 & 8 & 16 \\ 4 & 8 & 16 & 32 \end{pmatrix}$ a moins de 4 pivots : elle n'est pas inversible.

$$8) \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 3 & 1 & 0 & 0 & 0 \\ -2 & 1 & -5 & -6 & 0 & 1 & 0 & 0 \\ -5 & -10 & 9 & -15 & 0 & 0 & 1 & 0 \\ -6 & -12 & 27 & -18 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} L_2 \rightarrow L_2 + 2L_1 \\ L_3 \rightarrow L_3 + 5L_1 \\ L_4 \rightarrow L_4 + 6L_1 \\ \implies \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 5 & -9 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 15 & 0 & 6 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} L_4 \rightarrow L_4 + 15L_3 \\ \implies \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 5 & -9 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 81 & 0 & 15 & 1 \end{array} \right)$$

La matrice $\begin{pmatrix} 1 & 2 & -2 & 3 \\ -2 & 1 & -5 & -6 \\ -5 & -10 & 9 & -15 \\ -6 & -12 & 27 & -18 \end{pmatrix}$ a moins de 4 pivots : elle n'est pas inversible.

$$9) \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 3 & 1 & 0 & 0 & 0 \\ -2 & 1 & -5 & -6 & 0 & 1 & 0 & 0 \\ -5 & -10 & 9 & -15 & 0 & 0 & 1 & 0 \\ -6 & -12 & 12 & -19 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} L_2 \rightarrow L_2 + 2L_1 \\ L_3 \rightarrow L_3 + 5L_1 \\ L_4 \rightarrow L_4 + 6L_1 \\ \implies \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 5 & -9 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 6 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} L_1 \rightarrow L_1 + 3L_4 \\ \implies \end{array} \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 0 & 19 & 0 & 0 & 3 \\ 0 & 5 & -9 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 6 & 0 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} L_1 \rightarrow L_1 - 2L_3 \\ L_2 \rightarrow L_2 - 9L_3 \\ \implies \end{array} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 9 & 0 & -2 & 3 \\ 0 & 5 & 0 & 0 & -43 & 1 & -9 & 0 \\ 0 & 0 & -1 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 6 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} L_1 \rightarrow 5L_1 - 2L_2 \\ \implies \end{array}$$

$$\left(\begin{array}{cccc|cccc} 5 & 0 & 0 & 0 & 131 & -2 & 8 & 15 \\ 0 & 5 & 0 & 0 & -43 & 1 & -9 & 0 \\ 0 & 0 & -1 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 6 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} L_1 \rightarrow 1/5 L_1 \\ L_2 \rightarrow 1/5 L_2 \\ L_3 \rightarrow -L_3 \\ L_4 \rightarrow -L_4 \\ \implies \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{131}{5} & -\frac{2}{5} & \frac{8}{5} & 3 \\ 0 & 1 & 0 & 0 & -\frac{43}{5} & \frac{1}{5} & -\frac{9}{5} & 0 \\ 0 & 0 & 1 & 0 & -5 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -6 & 0 & 0 & -1 \end{array} \right)$$

$$\begin{pmatrix} 1 & 2 & -2 & 3 \\ -2 & 1 & -5 & -6 \\ -5 & -10 & 9 & -15 \\ -6 & -12 & 12 & -19 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{131}{5} & -\frac{2}{5} & \frac{8}{5} & 3 \\ -\frac{43}{5} & \frac{1}{5} & -\frac{9}{5} & 0 \\ -5 & 0 & -1 & 0 \\ -6 & 0 & 0 & -1 \end{pmatrix}$$