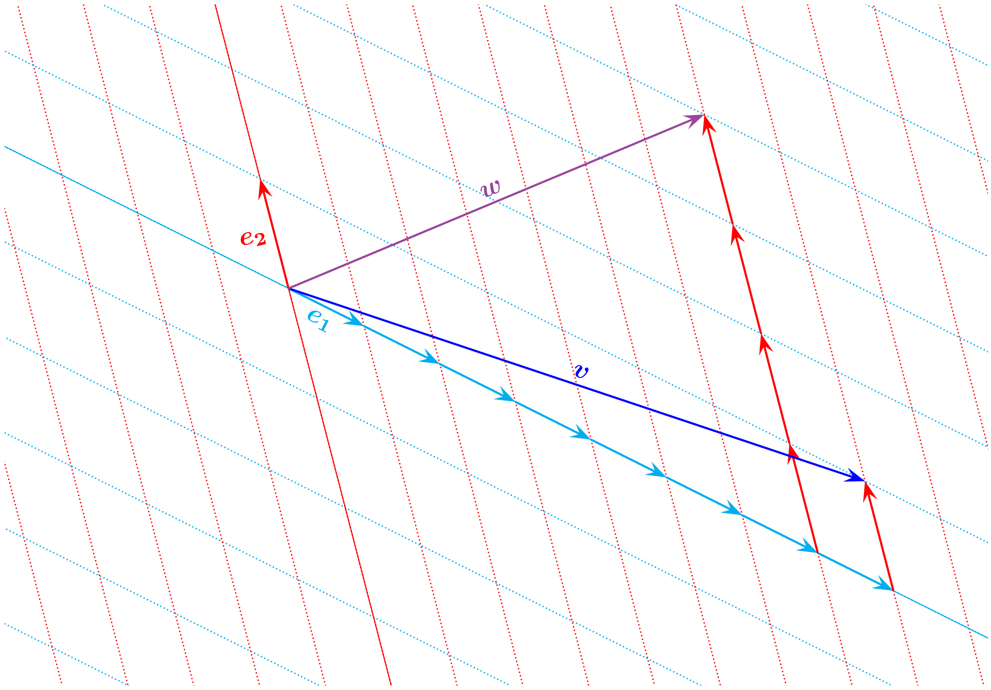
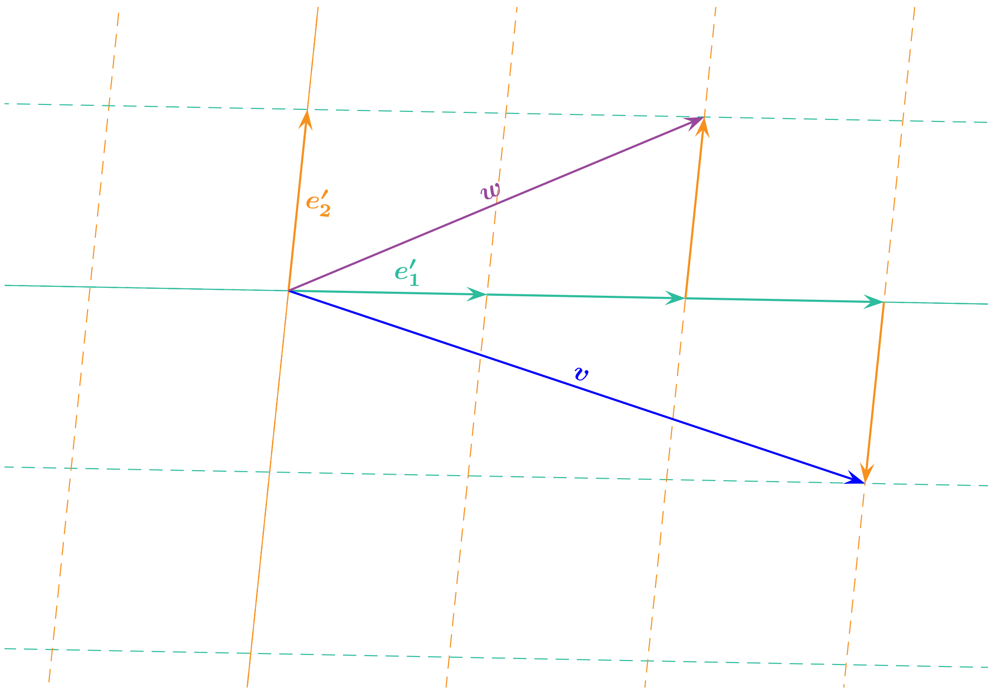
6.1 1) (a)



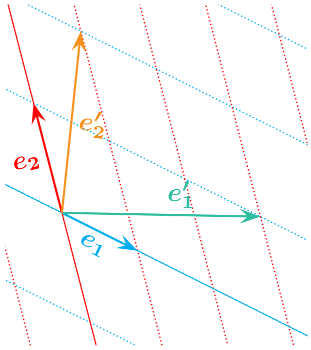
$V = \begin{pmatrix} 8 \\ 1 \end{pmatrix}_{\mathscr{B}} \qquad W = \begin{pmatrix} 7 \\ 4 \end{pmatrix}_{\mathscr{B}}$

(b)



$V' = \begin{pmatrix} 3 \\ -1 \end{pmatrix}_{\mathscr{B}'} \qquad W' = \begin{pmatrix} 2 \\ 1 \end{pmatrix}_{\mathscr{B}'}$

2) (a)



$$e_1' = 3\,e_1 + 1\,e_2 \qquad \mathrm{P} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$
$$e_2' = 1\,e_1 + 2\,e_2$$

(b) $\mathrm{V} = \mathrm{P}\,\mathrm{V}' = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix}_{\mathscr{B}'} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}_{\mathscr{B}}$

$\mathrm{W} = \mathrm{P}\,\mathrm{W}' = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}_{\mathscr{B}'} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}_{\mathscr{B}}$

(c) Calculons l'inverse de la matrice de passage :

$$\left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \xRightarrow{\mathrm{L}_1 \leftrightarrow \mathrm{L}_2} \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 3 & 1 & 1 & 0 \end{array}\right) \xRightarrow{\mathrm{L}_2 \to \mathrm{L}_2 - 3\,\mathrm{L}_1} \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -5 & 1 & -3 \end{array}\right)$$

$$\xRightarrow{\mathrm{L}_1 \to 5\,\mathrm{L}_1 + 2\,\mathrm{L}_2} \left(\begin{array}{cc|cc} 5 & 0 & 2 & -1 \\ 0 & -5 & 1 & -3 \end{array}\right) \xRightarrow[\mathrm{L}_2 \to -1/5\,\mathrm{L}_2]{\mathrm{L}_1 \to 1/5\,\mathrm{L}_1} \left(\begin{array}{cc|cc} 1 & 0 & \frac{2}{5} & -\frac{1}{5} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} \end{array}\right)$$

$$\mathrm{V}' = \mathrm{P}^{-1}\,\mathrm{V} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 1 \end{pmatrix}_{\mathscr{B}} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 8 \\ 1 \end{pmatrix}_{\mathscr{B}} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}_{\mathscr{B}'}$$

$$\mathrm{W}' = \mathrm{P}^{-1}\,\mathrm{W} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 7 \\ 4 \end{pmatrix}_{\mathscr{B}} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 7 \\ 4 \end{pmatrix}_{\mathscr{B}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}_{\mathscr{B}'}$$