6.2

$$e'_1 = \begin{pmatrix}1\\1\\0\end{pmatrix} = 1\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\begin{pmatrix}0\\0\\1\end{pmatrix} = 1\,\boldsymbol{e_1} + 1\,\boldsymbol{e_2} + 0\,\boldsymbol{e_3}$$

$$e'_2 = \begin{pmatrix}0\\1\\1\end{pmatrix} = 0\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\begin{pmatrix}0\\0\\1\end{pmatrix} = 0\,\boldsymbol{e_1} + 1\,\boldsymbol{e_2} + 1\,\boldsymbol{e_3} \qquad \mathrm{P} = \begin{pmatrix}1 & 0 & 1\\1 & 1 & 0\\0 & 1 & 1\end{pmatrix}$$

$$e'_1 = \begin{pmatrix}1\\0\\1\end{pmatrix} = 1\begin{pmatrix}1\\0\\0\end{pmatrix} + 0\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\begin{pmatrix}0\\0\\1\end{pmatrix} = 1\,\boldsymbol{e_1} + 0\,\boldsymbol{e_2} + 1\,\boldsymbol{e_3}$$

Calculons l'inverse de la matrice de passage :

$$\left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\1 & 1 & 0 & 0 & 1 & 0\\0 & 1 & 1 & 0 & 0 & 1\end{array}\right) \overset{\mathrm{L}_2 \to \mathrm{L}_2 - \mathrm{L}_1}{\Longrightarrow} \left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\0 & 1 & -1 & -1 & 1 & 0\\0 & 1 & 1 & 0 & 0 & 1\end{array}\right) \overset{\mathrm{L}_3 \to \mathrm{L}_3 - \mathrm{L}_2}{\Longrightarrow}$$

$$\left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\0 & 1 & -1 & -1 & 1 & 0\\0 & 0 & 2 & 1 & -1 & 1\end{array}\right) \overset{\substack{\mathrm{L}_1 \to 2\,\mathrm{L}_1 - \mathrm{L}_3\\ \mathrm{L}_2 \to 2\,\mathrm{L}_2 + \mathrm{L}_3}}{\Longrightarrow} \left(\begin{array}{ccc|ccc}2 & 0 & 0 & 1 & 1 & -1\\0 & 2 & 0 & -1 & 1 & 1\\0 & 0 & 2 & 1 & -1 & 1\end{array}\right) \overset{\substack{\mathrm{L}_1 \to 1/2\,\mathrm{L}_1\\ \mathrm{L}_2 \to 1/2\,\mathrm{L}_2\\ \mathrm{L}_3 \to 1/2\,\mathrm{L}_3}}{\Longrightarrow}$$

$$\left(\begin{array}{ccc|ccc}1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\end{array}\right) \qquad \text{On a trouvé } \mathrm{P}^{-1} = \begin{pmatrix}\frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\-\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\end{pmatrix}.$$

Il ne reste plus qu'à calculer les composantes du vecteur $\boldsymbol{v}$ dans la base $\mathscr{B}'$:

$$\mathrm{V}' = \mathrm{P}^{-1}\,\mathrm{V} = \begin{pmatrix}\frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\-\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}3\\-5\\10\end{pmatrix}_{\mathscr{B}} = \begin{pmatrix}-6\\1\\9\end{pmatrix}_{\mathscr{B}'}$$