5.1

1) $(x-5)^2+(y+2)^2=25$

$(x-5)^2+\big(y-(-2)\big)^2=5^2$

$C(5\,;-2) \qquad r=5$

2) $(x+2)^2+y^2=64$

$\big(x-(-2)\big)^2+(y-0)^2=8^2$

$C(-2\,;0) \qquad r=8$

3) $(x+5)^2+(y-2)^2=0$

$\big(x-(-5)\big)^2+(y-2)^2=0^2$

$C(-5\,;2) \qquad r=0$: cercle-point

4) $x^2+(y-5)^2=5$

$(x-0)^2+(y-5)^2=(\sqrt{5})^2$

$C(0\,;5) \qquad r=\sqrt{5}$

5) $x^2+y^2-2\,x+4\,y=20$

$x^2-2\,x+y^2+4\,y=20$

$\underbrace{x^2-2\,x+1}_{(x-1)^2}-1+\underbrace{y^2+4\,y+4}_{(y+2)^2}-4=20$

$(x-1)^2-1+(y+2)^2-4=20$

$(x-1)^2+(y+2)^2=25$

$(x-1)^2+\big(y-(-2)\big)^2=5^2$

$C(1\,;-2) \qquad r=5$

6) $x^2+y^2-2\,x+4\,y+14=0$

$x^2-2\,x+y^2+4\,y+14=0$

$\underbrace{x^2-2\,x+1}_{(x-1)^2}-1+\underbrace{y^2+4\,y+4}_{(y+2)^2}-4+14=0$

$(x-1)^2+(y+2)^2=-9$

Puisque $(x-1)^2+(y+2)^2\geqslant 0>-9$ quelles que soient les valeurs de x et y, cette équation n'est jamais vérifiée et correspond à la figure vide.

7) $x^2+y^2+4\,x-2\,y+5=0$

$x^2+4\,x+y^2-2\,y+5=0$

$\underbrace{x^2+4\,x+4}_{(x+2)^2}-4+\underbrace{y^2-2\,y+1}_{(y-1)^2}-1+5=0$

$(x+2)^2+(y-1)^2=0$

$\big(x-(-2)\big)^2+(y-1)^2=0^2$

$C(-2\,;1) \qquad r=0$: cercle-point

8) $x^2 + y^2 + x = 0$

$x^2 + x + y^2 = 0$

$\underbrace{x^2 + x + \frac{1}{4}}_{(x+\frac{1}{2})^2} - \frac{1}{4} + y^2 = 0$

$(x + \frac{1}{2})^2 + y^2 = \frac{1}{4}$

$\big(x - (-\frac{1}{2})\big)^2 + (y - 0)^2 = \big(\frac{1}{2}\big)^2$

$\mathrm{C}(-\frac{1}{2}\,;0) \qquad r = \frac{1}{2}$

9) $x^2 + y^2 + 6\,x - 4\,y + 14 = 0$

$x^2 + 6\,x + y^2 - 4\,y + 14 = 0$

$\underbrace{x^2 + 6\,x + 9}_{(x+3)^2} - 9 + \underbrace{y^2 - 4\,y + 4}_{(y-2)^2} - 4 + 14 = 0$

$(x + 3)^2 + (y - 2)^2 = -1$

Étant donné que $(x + 3)^2 + (y - 2)^2 \geqslant 0 > -1$ quels que soient les nombres x et y, cette équation correspond à la figure vide.

10) $x^2 + y^2 + y = 0$

$x^2 + \underbrace{y^2 + y + \frac{1}{4}}_{(y+\frac{1}{2})^2} - \frac{1}{4} = 0$

$x^2 + (y + \frac{1}{2})^2 = \frac{1}{4}$

$(x - 0)^2 + \big(y - (-\frac{1}{2})\big)^2 = \big(\frac{1}{2}\big)^2$

$\mathrm{C}(0\,; -\frac{1}{2}) \qquad r = \frac{1}{2}$

11) $80\,x^2 + 80\,y^2 - 120\,x + 80\,y = -17$

$x^2 + y^2 - \underbrace{\frac{120}{80}\,x}_{\frac{3}{2}\,x} + y = -\frac{17}{80}$

$\underbrace{x^2 - \frac{3}{2}\,x + \frac{9}{16}}_{(x-\frac{3}{4})^2} - \frac{9}{16} + \underbrace{y^2 + y + \frac{1}{4}}_{(y+\frac{1}{2})^2} - \frac{1}{4} = -\frac{17}{80}$

$(x - \frac{3}{4})^2 + (y + \frac{1}{2})^2 = -\frac{17}{80} + \frac{9}{16} + \frac{1}{4} = \frac{3}{5}$

$\mathrm{C}(\frac{3}{4}\,; -\frac{1}{2}) \qquad r = \sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}\,\sqrt{5}}{5} = \frac{\sqrt{15}}{5}$

12) $144\,x^2 + 144\,y^2 - 216\,x + 192\,y = -145$

$x^2 + y^2 - \underbrace{\frac{216}{144}\,x}_{\frac{3}{2}\,x} + \underbrace{\frac{192}{144}\,y}_{\frac{4}{3}\,y} = -\frac{145}{144}$

$\underbrace{x^2 - \frac{3}{2}\,x + \frac{9}{16}}_{(x-\frac{3}{4})^2} - \frac{9}{16} + \underbrace{y^2 + \frac{4}{3}\,y + \frac{4}{9}}_{(y+\frac{2}{3})^2} - \frac{4}{9} = -\frac{145}{144}$

$(x - \frac{3}{4})^2 + (y + \frac{2}{3})^2 = -\frac{145}{144} + \frac{9}{16} + \frac{4}{9} = 0$

$\mathrm{C}(\frac{3}{4}\,; -\frac{2}{3}) \qquad r = 0$: cercle-point