4.4 1) $z\,\overline{z} = (a + b\,i)\,(a - b\,i) = a^2 - a\,b\,i + a\,b\,i - b^2\,i^2 = a^2 + b^2$

2) $\overline{\overline{z}} = \overline{\overline{a + b\,i}} = \overline{a - b\,i} = a - (-b)\,i = a + b\,i = z$

3) $\overline{z_1 + z_2} = \overline{(a_1 + b_1\,i) + (a_2 + b_2\,i)} = \overline{(a_1 + a_2) + (b_1 + b_2)\,i} =$
$(a_1 + a_2) - (b_1 + b_2)\,i = (a_1 - b_1\,i) + (a_2 - b_2\,i) = \overline{z_1} + \overline{z_2}$

4) $\overline{z_1 - z_2} = \overline{(a_1 + b_1\,i) - (a_2 + b_2\,i)} = \overline{(a_1 - a_2) + (b_1 - b_2)\,i} =$
$(a_1 - a_2) - (b_1 - b_2)\,i = (a_1 - b_1\,i) - (a_2 - b_2\,i) = \overline{z_1} - \overline{z_2}$

5) $\overline{z_1\,z_2} = \overline{(a_1 + b_1\,i)\,(a_2 + b_2\,i)} = \overline{a_1\,a_2 + a_1\,b_2\,i + a_2\,b_1\,i + b_1\,b_2\,i^2} =$
$\overline{(a_1\,a_2 - b_1\,b_2) + (a_1\,b_2 + a_2\,b_1)\,i} = (a_1\,a_2 - b_1\,b_2) - (a_1\,b_2 + a_2\,b_1)\,i$

$\overline{z_1}\,\overline{z_2} = \overline{(a_1 + b_1\,i)}\,\overline{(a_2 + b_2\,i)} = (a_1 - b_1\,i)\,(a_2 - b_2\,i) =$
$a_1\,a_2 - a_1\,b_2\,i - a_2\,b_1\,i + b_1\,b_2\,i^2 = (a_1\,a_2 - b_1\,b_2) - (a_1\,b_2 + a_2\,b_1)\,i$

En définitive $\overline{z_1\,z_2} = (a_1\,a_2 - b_1\,b_2) - (a_1\,b_2 + a_2\,b_1)\,i = \overline{z_1}\,\overline{z_2}$

6) Utilisons la propriété $\overline{z_1}\,\overline{z_2} = \overline{z_1\,z_2}$ avec $z_1 = z$ et $z_2 = \dfrac{1}{z}$:

$\overline{z} \cdot \overline{\left(\dfrac{1}{z}\right)} = \overline{z \cdot \dfrac{1}{z}} = \overline{1} = 1$

En divisant cette égalité par $\overline{z}$, on obtient : $\overline{\left(\dfrac{1}{z}\right)} = \dfrac{1}{\overline{z}}$.

7) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \overline{z_1\,\left(\dfrac{1}{z_2}\right)} = \overline{z_1}\,\overline{\left(\dfrac{1}{z_2}\right)} = \overline{z_1}\,\dfrac{1}{\overline{z_2}} = \dfrac{\overline{z_1}}{\overline{z_2}}$