5.11 1) $|z_1| = |1+\sqrt{3}\,i| = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$z_1 = 1+\sqrt{3}\,i = 2\left(\frac{1}{2}+i\,\frac{\sqrt{3}}{2}\right) = 2\left(\cos(\frac{\pi}{3})+i\,\sin(\frac{\pi}{3})\right)$

$|z_2| = |\frac{1}{2}-\frac{1}{2}\,i| = |\frac{1}{2}\,(1-i)| = |\frac{1}{2}|\,|1-i| = \frac{1}{2}\,\sqrt{1^2+(-1)^2} = \frac{1}{2}\,\sqrt{2} = \frac{\sqrt{2}}{2}$

$z_2 = \frac{1}{2}-\frac{1}{2}\,i = \frac{\sqrt{2}}{2}\left(\frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}}+i\left(-\frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}}\right)\right) = \frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}+i(-\frac{1}{\sqrt{2}})\right) =$

$\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}+i(-\frac{\sqrt{2}}{2})\right) = \frac{\sqrt{2}}{2}\left(\cos(\frac{7\pi}{4})+i\,\sin(\frac{7\pi}{4})\right)$

2) (a) Forme algébrique

$z_1\,z_2 = (1+\sqrt{3}\,i)\,(\frac{1}{2}-\frac{1}{2}\,i) = \frac{1}{2}-\frac{1}{2}\,i+\frac{\sqrt{3}}{2}\,i-\frac{\sqrt{3}}{2}\,i^2 = \frac{\sqrt{3}+1}{2}+\frac{\sqrt{3}-1}{2}\,i$

(b) Forme trigonométrique

$z_1\,z_2 = 2\left(\cos(\frac{\pi}{3})+i\,\sin(\frac{\pi}{3})\right)\cdot\frac{\sqrt{2}}{2}\left(\cos(\frac{7\pi}{4})+i\,\sin(\frac{7\pi}{4})\right) =$

$2\cdot\frac{\sqrt{2}}{2}\left(\cos(\frac{\pi}{3}+\frac{7\pi}{4})+i\,\sin(\frac{\pi}{3}+\frac{7\pi}{4})\right) = \sqrt{2}\left(\cos(\frac{25\pi}{12})+i\,\sin(\frac{25\pi}{12})\right) =$

$\sqrt{2}\left(\cos(\frac{\pi}{12}+2\pi)+i\,\sin(\frac{\pi}{12}+2\pi)\right) = \sqrt{2}\left(\cos(\frac{\pi}{12})+i\,\sin(\frac{\pi}{12})\right)$

3) L'égalité $\frac{\sqrt{3}+1}{2}+\frac{\sqrt{3}-1}{2}\,i = \sqrt{2}\left(\cos(\frac{\pi}{12})+i\,\sin(\frac{\pi}{12})\right)$ donne

(a) $\frac{\sqrt{3}+1}{2} = \sqrt{2}\,\cos(\frac{\pi}{12})$

$\cos(\frac{\pi}{12}) = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{(\sqrt{3}+1)\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

(b) $\frac{\sqrt{3}-1}{2} = \sqrt{2}\,\sin(\frac{\pi}{12})$

$\sin(\frac{\pi}{12}) = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{(\sqrt{3}-1)\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$