5.8

1) $z_1\,z_2 = r_1\big(\cos(\varphi_1) + i\,\sin(\varphi_1)\big)\cdot r_2\big(\cos(\varphi_2) + i\,\sin(\varphi_2)\big) =$

$r_1\,r_2\,\Big(\cos(\varphi_1)\,\cos(\varphi_2) - \sin(\varphi_1)\,\sin(\varphi_2) + i\,\big(\sin(\varphi_1)\,\cos(\varphi_2) + \cos(\varphi_1)\,\sin(\varphi_2)\big)\Big) =$

$r_1\,r_2\,\big(\cos(\varphi_1 + \varphi_2) + i\,\sin(\varphi_1 + \varphi_2)\big)$

Ainsi $\quad |z_1\,z_2| = r_1\,r_2 = |z_1|\,|z_2| \quad$ et $\quad \arg(z_1\,z_2) = \varphi_1 + \varphi_2 = \arg(z_1) + \arg(z_2)$

2) Posons $z' = \dfrac{1}{r}\big(\cos(\varphi) - i\,\sin(\varphi)\big) = \dfrac{1}{r}\big(\cos(-\varphi) + i\,\sin(-\varphi)\big)$.

On a $\quad |z'| = \dfrac{1}{r} = \dfrac{1}{|z|} \quad$ et $\quad \arg(z') = -\varphi = -\arg(z)$.

De plus, $z\,z' = r\big(\cos(\varphi) + i\,\sin(\varphi)\big)\cdot\dfrac{1}{r}\big(\cos(\varphi) - i\,\sin(\varphi)\big) =$

$r\cdot\dfrac{1}{r}\big(\cos(\varphi - \varphi) + i\,\sin(\varphi - \varphi)\big) = 1\,\big(\cos(0) + i\,\sin(0)\big) = 1 + i\cdot 0 = 1$

ce qui montre que $z' = \dfrac{1}{z}$.

3) $\left|\dfrac{z_1}{z_2}\right| = \left|z_1\cdot\dfrac{1}{z_2}\right| = |z_1|\,\left|\dfrac{1}{z_2}\right| = |z_1|\,\dfrac{1}{|z_2|} = \dfrac{|z_1|}{|z_2|} = \dfrac{r_1}{r_2}$

$\arg\left(\dfrac{z_1}{z_2}\right) = \arg\left(z_1\cdot\dfrac{1}{z_2}\right) = \arg(z_1) + \arg\left(\dfrac{1}{z_2}\right) = \arg(z_1) + \big(-\arg(z_2)\big) =$

$\arg(z_1) - \arg(z_2) = \varphi_1 - \varphi_2$