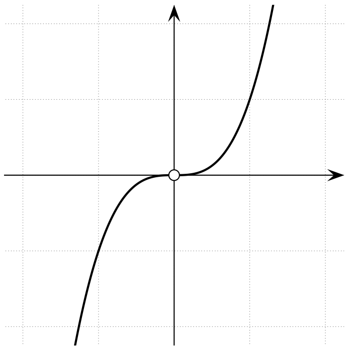
6.10 1) $f'(x) = 3\,x^2$

$f''(x) = 6\,x$

		0	
$6\,x$	$-$	0	$+$
f''	$-$	0	$+$
f	$\frown$	infl	$\smile$

$f(0) = 0^3 = 0$

Le point $(0\,;0)$ est un point d'inflexion.



2) $f(x) = \dfrac{x^3-8}{x} = x^2 - \dfrac{8}{x} = x^2 - 8\,x^{-1}$

$f'(x) = 2\,x + 8\,x^{-2}$

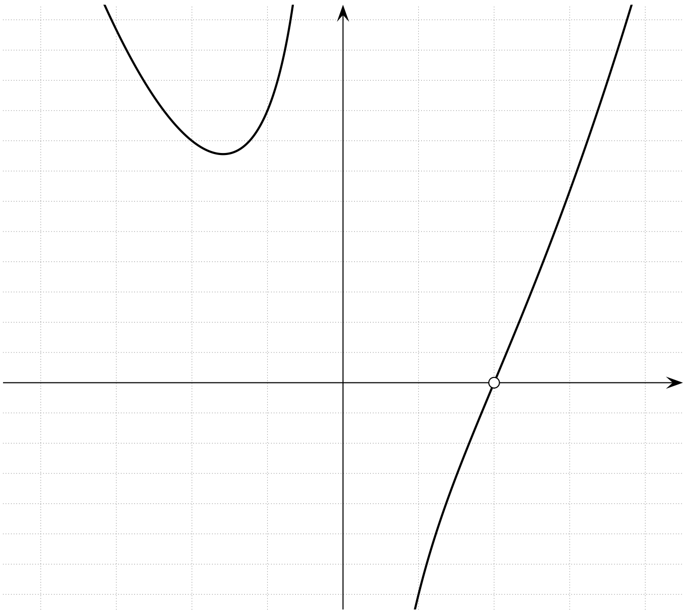
$f''(x) = 2 - 16\,x^{-3} = 2 - \dfrac{16}{x^3} = \dfrac{2\,x^3-16}{x^3} = \dfrac{2\,(x^3-8)}{x^3}$

$= \dfrac{2\,(x-2)\,(x^2+2\,x+4)}{x^3}$

		0		2	
2	$+$	‖	$+$		$+$
$x-2$	$-$	‖	$-$	0	$+$
$x^2+2\,x+4$	$+$	‖	$+$		$+$
x^3	$-$	‖	$+$		$+$
f''	$+$	‖	$-$	0	$+$
f	$\smile$	‖	$\frown$	infl	$\smile$

$f(2) = \dfrac{2^3-8}{2} = 0$

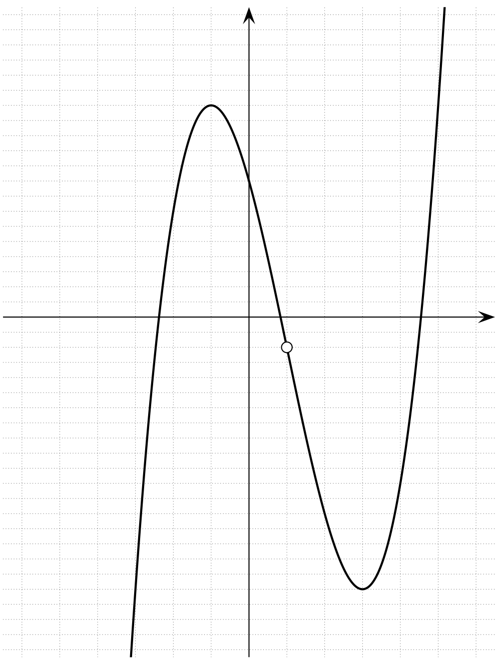
Le point $(0\,;2)$ est un point d'inflexion.



3) $f'(x) = 3\,x^2 - 6\,x - 9$
$f''(x) = 6\,x - 6 = 6\,(x - 1)$

		1	
6	$+$		$+$
$x - 1$	$-$	0	$+$
f''	$-$	0	$+$
f	$\frown$	infl	$\smile$

$f(1) = 1^3 - 3 \cdot 1^2 - 9 \cdot 1 + 9 = -2$
Le point $(1\,;-2)$ est un point d'inflexion.



4) $f'(x) = \left(\dfrac{x}{x^2+3}\right)' = \dfrac{(x)'\,(x^2+3) - x\,(x^2+3)'}{(x^2+3)^2} = \dfrac{1\,(x^2+3) - x \cdot 2\,x}{(x^2+3)^2}$

$$= \frac{3-x^2}{(x^2+3)^2}$$

$$f''(x) = \left(\frac{3-x^2}{(x^2+3)^2}\right)' = \frac{(3-x^2)'\,(x^2+3)^2 - (3-x^2)\,\big((x^2+3)^2\big)'}{\big((x^2+3)^2\big)^2}$$

$$= \frac{-2\,x\,(x^2+3)^2 - (3-x^2)\,2\,(x^2+3)\,\overbrace{(x^2+3)'}^{2\,x}}{(x^2+3)^4}$$

$$= \frac{-2\,x\,(x^2+3)^2 - 4\,x\,(3-x^2)\,(x^2+3)}{(x^2+3)^4}$$

$$= \frac{-2\,x\,(x^2+3)\,\big((x^2+3) + 2\,(3-x^2)\big)}{(x^2+3)^4} = \frac{-2\,x\,(-x^2+9)}{(x^2+3)^3}$$

$$= \frac{2\,x\,(x^2-9)}{(x^2+3)^3} = \frac{2\,x\,(x+3)\,(x-3)}{(x^2+3)^3}$$

		-3		0		3	
$2\,x$	$-$		$-$	0	$+$		$+$
$x+3$	$-$	0	$+$		$+$		$+$
$x-3$	$-$		$-$		$-$	0	$+$
$(x^2+3)^3$	$+$		$+$		$+$		$+$
f''	$-$	0	$+$	0	$-$	0	$+$
f	$\frown$	infl	$\smile$	infl	$\frown$	infl	$\smile$

$f(-3) = \frac{-3}{(-3)^2+3} = -\frac{1}{4}$
Le point $(-3\,;-\frac{1}{4})$ est un point d'inflexion.

$f(0) = \frac{0}{0^2+3} = 0$
Le point $(0\,;0)$ est un point d'inflexion.

$f(3) = \frac{3}{3^2+3} = \frac{1}{4}$
Le point $(3\,;\frac{1}{4})$ est un point d'inflexion.