**6.11** 1) $f(x) = \frac{1}{4}\,x^2 + x + 1 = \frac{1}{4}\,(x^2 + 4\,x + 4) = \frac{1}{4}\,(x+2)^2$

| | | $-2$ | |
|---|---|---|---|
| $\frac{1}{4}$ | $+$ | | $+$ |
| $(x+2)^2$ | $+$ | $0$ | $+$ |
| $f$ | $+$ | $0$ | $+$ |

$f'(x) = \frac{1}{4}\,(x^2 + 4\,x + 4)' = \frac{1}{4}\,(2\,x + 4) = \frac{1}{2}\,(x+2)$

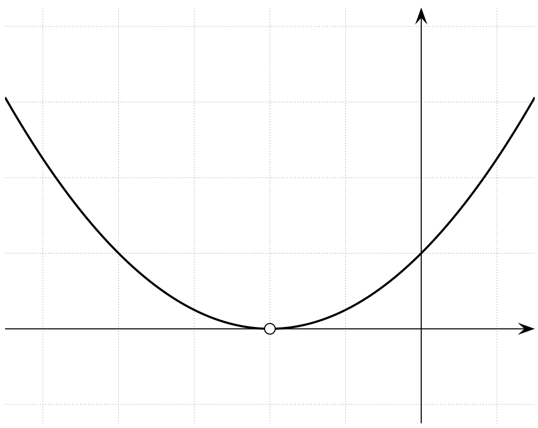
| | | $-2$ | |
|---|---|---|---|
| $\frac{1}{2}$ | $+$ | | $+$ |
| $x+2$ | $-$ | $0$ | $+$ |
| $f'$ | $-$ | $0$ | $+$ |
| $f$ | $\searrow$ | min | $\nearrow$ |

$f(-2) = \frac{1}{4}\,\big((-2)+2\big)^2 = 0$

Le point $(-2\,;0)$ est un minimum (absolu).

$f''(x) = \frac{1}{2}\,(x+2)' = \frac{1}{2} \cdot 1 = \frac{1}{2}$

| | |
|---|---|
| $\frac{1}{2}$ | $+$ |
| $f''$ | $+$ |
| $f$ | $\smile$ |



2) $f(x) = -x^2 + x + 2 = -(x^2 - x - 2) = -(x-2)\,(x+1) = (2-x)\,(x+1)$

| | | $-1$ | | $2$ | |
|---|---|---|---|---|---|
| $2-x$ | $+$ | | $+$ | $0$ | $-$ |
| $x+1$ | $-$ | $0$ | $+$ | | $+$ |
| $f$ | $-$ | $0$ | $+$ | $0$ | $-$ |

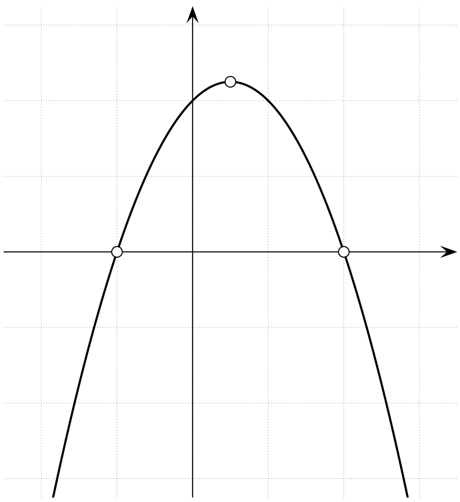
$f'(x) = -2\,x + 1$

| | | $\frac{1}{2}$ | |
|---|---|---|---|
| $-2\,x+1$ | $+$ | $0$ | $-$ |
| $f'$ | $+$ | $0$ | $-$ |
| $f$ | $\nearrow$ | max | $\searrow$ |

$f(\frac{1}{2}) = -(\frac{1}{2})^2 + \frac{1}{2} + 2 = \frac{9}{4}$
Le point $(\frac{1}{2}\,;\frac{9}{4})$ est un maximum (absolu).

$f''(x) = -2$

| | |
|---|---|
| $-2$ | $-$ |
| $f''$ | $-$ |
| $f$ | $\frown$ |



3) $f(x) = x^3 - 3\,x = x\,(x^2 - 3) = x\,(x + \sqrt{3})\,(x - \sqrt{3})$

| | | $-\sqrt{3}$ | | $0$ | | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|
| $x$ | $-$ | | $-$ | $0$ | $+$ | | $+$ |
| $x + \sqrt{3}$ | $-$ | $0$ | $+$ | | $+$ | | $+$ |
| $x - \sqrt{3}$ | $-$ | | $-$ | | $-$ | $0$ | $+$ |
| $f$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

$f'(x) = 3\,x^2 - 3 = 3\,(x^2 - 1) = 3\,(x + 1)\,(x - 1)$

| | | $-1$ | | $1$ | |
|---|---|---|---|---|---|
| $3$ | $+$ | | $+$ | | $+$ |
| $x + 1$ | $-$ | $0$ | $+$ | | $+$ |
| $x - 1$ | $-$ | | $-$ | $0$ | $+$ |
| $f'$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f$ | $\nearrow$ | max | $\searrow$ | min | $\nearrow$ |

$f(-1) = (-1)^3 - 3 \cdot (-1) = 2$
Le point $(-1\,;2)$ est un maximum (local).

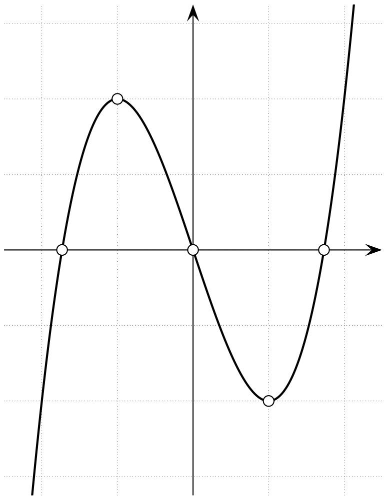
$f(1) = 1^3 - 3 \cdot 1 = -2$
Le point $(1\,;-2)$ est un minimum (local).

$f''(x) = 6\,x$

| | | 0 | |
|---|---|---|---|
| $6\,x$ | $-$ | 0 | $+$ |
| $f''$ | $-$ | 0 | $+$ |
| $f$ | $\frown$ | infl | $\smile$ |

$f(0) = 0^3 - 3 \cdot 0 = 0$

Le point $(0\,;0)$ est un point d'inflexion.



4) $f(x) = 3\,x^4 + 4\,x^3 = x^3\,(3\,x + 4)$

| | | $-\frac{4}{3}$ | | 0 | |
|---|---|---|---|---|---|
| $x^3$ | $-$ | | $-$ | 0 | $+$ |
| $3\,x + 4$ | $-$ | 0 | $+$ | | $+$ |
| $f$ | $+$ | 0 | $-$ | 0 | $+$ |

$f'(x) = 12\,x^3 + 12\,x^2 = 12\,x^2\,(x + 1)$

| | | $-1$ | | 0 | |
|---|---|---|---|---|---|
| 12 | $+$ | | $+$ | | $+$ |
| $x^2$ | $+$ | | $+$ | 0 | $+$ |
| $x + 1$ | $-$ | 0 | $+$ | | $+$ |
| $f'$ | $-$ | 0 | $+$ | 0 | $+$ |
| $f$ | $\searrow$ | min | $\nearrow$ | replat | $\nearrow$ |

$f(-1) = 3 \cdot (-1)^4 + 4 \cdot (-1)^3 = -1$

Le point $(-1\,;-1)$ est un minimum (absolu).

$f(0) = 3 \cdot 0^4 + 4 \cdot 0^3 = 0$

Le point $(0\,;0)$ est un replat.

$f''(x) = 36\,x^2 + 24\,x = 12\,x\,(3\,x + 2)$

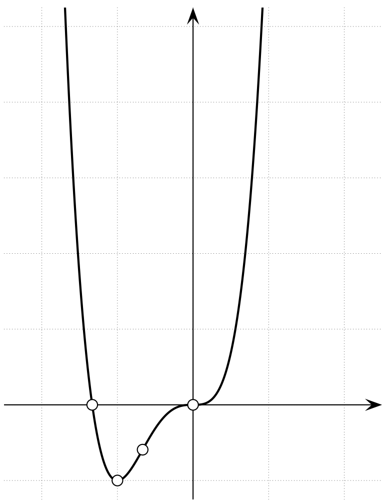
| | | $-\frac{2}{3}$ | | $0$ | |
|---|---|---|---|---|---|
| $12$ | $+$ | | $+$ | | $+$ |
| $x$ | $-$ | | $-$ | $0$ | $+$ |
| $3\,x+2$ | $-$ | $0$ | $+$ | | $+$ |
| $f''$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f$ | $\smile$ | infl | $\frown$ | infl | $\smile$ |

$f(-\frac{2}{3}) = 3 \cdot (-\frac{2}{3})^4 + 4 \cdot (-\frac{2}{3})^3 = \frac{48}{81} - \frac{32}{27} = -\frac{16}{27}$

Le point $(-\frac{2}{3}\,;-\frac{16}{27})$ est un point d'inflexion.

$f(0) = 3 \cdot 0^4 + 4 \cdot 0^3$

Le point $(0\,;0)$ est un point d'inflexion.



5) $f(x) = -\frac{1}{9}\,(x^2 - 2\,x + 1)\,(2\,x+7) = -\frac{1}{9}\,(x-1)^2\,(2\,x+7)$

| | | $-\frac{7}{2}$ | | $1$ | |
|---|---|---|---|---|---|
| $-\frac{1}{9}$ | $-$ | | $-$ | | $-$ |
| $(x-1)^2$ | $+$ | | $+$ | $0$ | $+$ |
| $2\,x+7$ | $-$ | $0$ | $+$ | | $+$ |
| $f$ | $+$ | $0$ | $-$ | $0$ | $-$ |

$$\begin{aligned}
f'(x) &= -\tfrac{1}{9}\,\big((x-1)^2\,(2\,x+7)\big)' \\
&= -\tfrac{1}{9}\,\Big(\big((x-1)^2\big)'\,(2\,x+7) + (x-1)^2\,(2\,x+7)'\Big) \\
&= -\tfrac{1}{9}\,\big(2\,(x-1)\,\underbrace{(x-1)'}_{1}\,(2\,x+7) + (x-1)^2\,2\big) \\
&= -\tfrac{1}{9}\,\cdot 2\,(x-1)\,\big((2\,x+7)+(x-1)\big) = -\tfrac{2}{9}\,(x-1)\,\underbrace{(3\,x+6)}_{3\,(x+2)} \\
&= -\tfrac{2}{3}\,(x-1)\,(x+2)
\end{aligned}$$

| | | $-2$ | | $1$ | |
|---|---|---|---|---|---|
| $-\frac{2}{3}$ | $-$ | | $-$ | | $-$ |
| $x-1$ | $-$ | | $-$ | $0$ | $+$ |
| $x+2$ | $-$ | $0$ | $+$ | | $+$ |
| $f'$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| $f$ | $\searrow$ | min | $\nearrow$ | max | $\searrow$ |

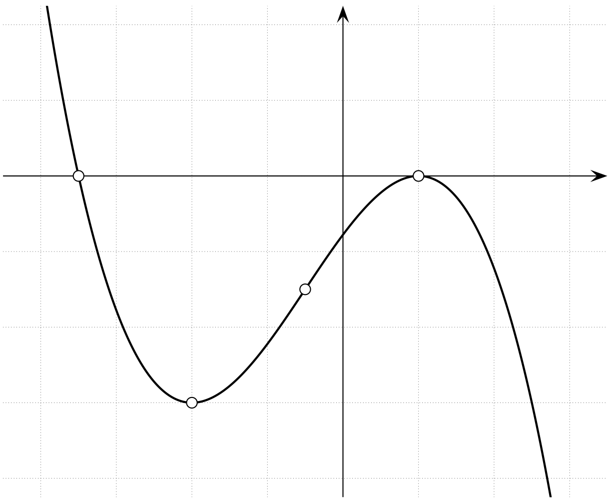
$f(-2) = -\frac{1}{9}\big((-2)^2 - 2\cdot(-2)+1\big)\big(2\cdot(-2)+7\big) = -3$
Le point $(-2\,;-3)$ est un minimum (local).

$f(1) = -\frac{1}{9}\,(1^2 - 2\cdot 1+1)\,(2\cdot 1+7) = 0$
Le point $(1\,;0)$ est un maximum (local).

$$f''(x) = -\frac{2}{3}\big((x-1)\,(x+2)\big)' = -\frac{2}{3}\big(\underbrace{(x-1)'}_{1}(x+2)+(x-1)\,\underbrace{(x+2)'}_{1}\big)$$
$$= -\frac{2}{3}\big((x+2)+(x-1)\big) = -\frac{2}{3}\,(2\,x+1)$$

| | | $-\frac{1}{2}$ | |
|---|---|---|---|
| $-\frac{2}{3}$ | $-$ | | $-$ |
| $2\,x+1$ | $-$ | $0$ | $+$ |
| $f''$ | $+$ | $0$ | $-$ |
| $f$ | $\smile$ | infl | $\frown$ |

$f(-\frac{1}{2}) = -\frac{1}{9}\big((-\frac{1}{2})^2 - 2\cdot(-\frac{1}{2})+1\big)\big(2\cdot(-\frac{1}{2})+7\big) = -\frac{3}{2}$
Le point $(-\frac{1}{2}\,;-\frac{3}{2})$ est un point d'inflexion.



6) $f(x) = \frac{1}{6}\,(2\,x^3 - 3\,x^2 - 12\,x + 18) = \frac{1}{6}\big(x^2\,(2\,x-3) - 6\,(2\,x-3)\big)$
$= \frac{1}{6}\,(2\,x-3)\,(x^2-6) = \frac{1}{6}\,(2\,x-3)\,(x+\sqrt{6})\,(x-\sqrt{6})$

| | | $-\sqrt{6}$ | | $\frac{3}{2}$ | | $\sqrt{6}$ | |
|---|---|---|---|---|---|---|---|
| $\frac{1}{6}$ | $+$ | | $+$ | | $+$ | | $+$ |
| $2\,x-3$ | $-$ | | $-$ | $0$ | $+$ | | $+$ |
| $x+\sqrt{6}$ | $-$ | $0$ | $+$ | | $+$ | | $+$ |
| $x-\sqrt{6}$ | $-$ | | $-$ | | $-$ | $0$ | $+$ |
| $f$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

$f'(x) = \frac{1}{6}\,(2\,x^3 - 3\,x^2 - 12\,x + 18)' = \frac{1}{6}\,(6\,x^2 - 6\,x - 12) = x^2 - x - 2$
$\quad = (x+1)\,(x-2)$

| | | $-1$ | | $2$ | |
|---|---|---|---|---|---|
| $x+1$ | $-$ | $0$ | $+$ | | $+$ |
| $x-2$ | $-$ | | $-$ | $0$ | $+$ |
| $f'$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f$ | $\nearrow$ | max | $\searrow$ | min | $\nearrow$ |

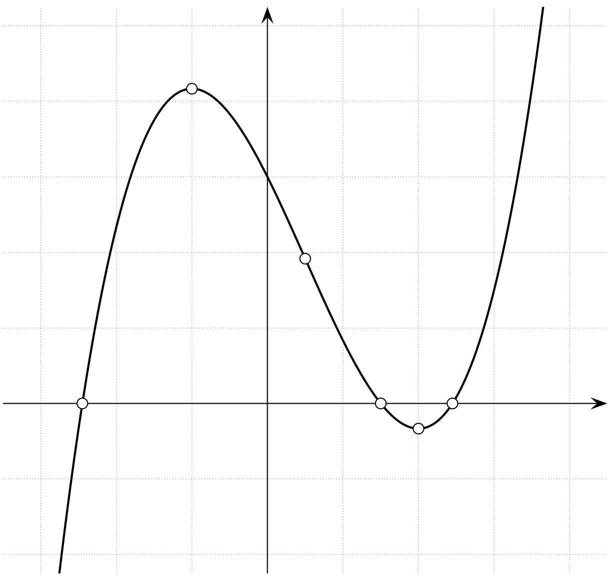
$f(-1) = \frac{1}{6}\big(2\cdot(-1)^3 - 3\cdot(-1)^2 - 12\cdot(-1) + 18\big) = \frac{25}{6}$
Le point $(-1\,;\frac{25}{6})$ est un maximum (local).

$f(2) = \frac{1}{6}\,(2\cdot 2^3 - 3\cdot 2^2 - 12\cdot 2 + 18) = -\frac{1}{3}$
Le point $(2\,;-\frac{1}{3})$ est un minimum (local).

$f''(x) = (x^2 - x - 2)' = 2\,x - 1$

| | | $\frac{1}{2}$ | |
|---|---|---|---|
| $2\,x-1$ | $-$ | $0$ | $+$ |
| $f''$ | $-$ | $0$ | $+$ |
| $f$ | $\frown$ | infl | $\smile$ |

$f(\frac{1}{2}) = \frac{1}{6}\big(2\cdot(\frac{1}{2})^3 - 3\cdot(\frac{1}{2})^2 - 12\cdot\frac{1}{2} + 18\big) = \frac{23}{12}$
Le point $(\frac{1}{2}\,;\frac{23}{12})$ est un point d'inflexion.



7) $f(x) = -\frac{1}{4}\,(x^4 - 6\,x^2 + 8) = -\frac{1}{4}\,(x^2-4)\,(x^2-2)$
$\quad = -\frac{1}{4}\,(x+2)\,(x-2)\,(x+\sqrt{2})\,(x-\sqrt{2})$

| | | $-2$ | | $-\sqrt{2}$ | | $\sqrt{2}$ | | $2$ | |
|---|---|---|---|---|---|---|---|---|---|
| $-\frac{1}{4}$ | $-$ | | $-$ | | $-$ | | $-$ | | $-$ |
| $x+2$ | $-$ | $0$ | $+$ | | $+$ | | $+$ | | $+$ |
| $x+\sqrt{2}$ | $-$ | | $-$ | $0$ | $+$ | | $+$ | | $+$ |
| $x-\sqrt{2}$ | $-$ | | $-$ | | $-$ | $0$ | $+$ | | $+$ |
| $x-2$ | $-$ | | $-$ | | $-$ | | $-$ | $0$ | $+$ |
| $f$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ | $0$ | $-$ |

$$f'(x) = -\tfrac{1}{4}\,(x^4 - 6\,x^2 + 8)' = -\tfrac{1}{4}\,(4\,x^3 - 12\,x) = -x\,(x^2 - 3)$$
$$= -x\,(x + \sqrt{3})\,(x - \sqrt{3})$$

| | | $-\sqrt{3}$ | | $0$ | | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|
| $-x$ | $+$ | | $+$ | $0$ | $-$ | | $-$ |
| $x+\sqrt{3}$ | $-$ | $0$ | $+$ | | $+$ | | $+$ |
| $x-\sqrt{3}$ | $-$ | | $-$ | | $-$ | $0$ | $+$ |
| $f'$ | $+$ | $0$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| $f$ | $\nearrow$ | max | $\searrow$ | min | $\nearrow$ | max | $\searrow$ |

$f(-\sqrt{3}) = -\frac{1}{4}\big((-\sqrt{3})^4 - 6\cdot(-\sqrt{3})^2 + 8\big) = \frac{1}{4}$
Le point $(-\sqrt{3}\,;\frac{1}{4})$ est un maximum (absolu).

$f(0) = -\frac{1}{4}\,(0^4 - 6\cdot 0^2 + 8) = -2$
Le point $(0\,;-2)$ est un minimum (local).

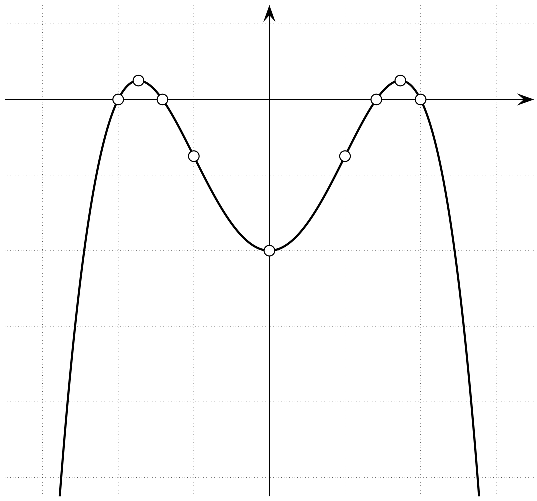
$f(\sqrt{3}) = -\frac{1}{4}\big((\sqrt{3})^4 - 6\cdot(\sqrt{3})^2 + 8\big) = \frac{1}{4}$
Le point $(\sqrt{3}\,;\frac{1}{4})$ est un maximum (absolu).

$$f''(x) = (-x^3 + 3\,x)' = -3\,x^2 + 3 = 3\,(1 - x^2) = 3\,(1+x)\,(1-x)$$

| | | $-1$ | | $1$ | |
|---|---|---|---|---|---|
| $3$ | $+$ | | $+$ | | $+$ |
| $1+x$ | $-$ | $0$ | $+$ | | $+$ |
| $1-x$ | $+$ | | $+$ | $0$ | $-$ |
| $f''$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| $f$ | $\frown$ | infl | $\smile$ | infl | $\frown$ |

$f(-1) = -\frac{1}{4}\big((-1)^4 - 6\cdot(-1)^2 + 8\big) = -\frac{3}{4}$
Le point $(-1\,;-\frac{3}{4})$ est un point d'inflexion.

$f(1) = -\frac{1}{4}\,(1^4 - 6\cdot 1^2 + 8) = -\frac{3}{4}$
Le point $(1\,;-\frac{3}{4})$ est un point d'inflexion.



8) $f(x) = 4\,x^3 - 3\,x + 1 = 0$ admet pour solutions entières possibles les diviseurs de 1, à savoir 1 et $-1$.

$f(1) = 4 \cdot 1^3 - 3 \cdot 1 + 1 = 2 \neq 0$
$f(-1) = 4 \cdot (-1)^3 - 3 \cdot (-1) + 1 = 0$

Factorisons $4\,x^3 - 3\,x + 1$ à l'aide du schéma de Horner :

| | | | |
|---|---|---|---|
| 4 | 0 | $-3$ | 1 |
| | $-4$ | 4 | $-1$ |
| 4 | $-4$ | 1 | 0 |

Ainsi $f(x) = 4\,x^3 - 3\,x + 1 = (x+1)\,(4\,x^2 - 4\,x + 1) = (x+1)\,(2\,x-1)^2$

| | | $-1$ | | $\frac{1}{2}$ | |
|---|---|---|---|---|---|
| $x+1$ | $-$ | 0 | $+$ | | $+$ |
| $(2\,x-1)^2$ | $+$ | | $+$ | 0 | $+$ |
| $f$ | $-$ | 0 | $+$ | 0 | $+$ |

$f'(x) = (4\,x^3 - 3\,x + 1)' = 12\,x^2 - 3 = 3\,(4\,x^2 - 1) = 3\,(2\,x+1)\,(2\,x-1)$

| | | $-\frac{1}{2}$ | | $\frac{1}{2}$ | |
|---|---|---|---|---|---|
| 3 | $+$ | | $+$ | | $+$ |
| $2\,x+1$ | $-$ | 0 | $+$ | | $+$ |
| $2\,x-1$ | $-$ | | $-$ | 0 | $+$ |
| $f'$ | $+$ | 0 | $-$ | 0 | $-$ |
| $f$ | $\nearrow$ | max | $\searrow$ | min | $\nearrow$ |

$f(-\frac{1}{2}) = 4 \cdot (-\frac{1}{2})^3 - 3 \cdot (-\frac{1}{2}) + 1 = 2$
Le point $(-\frac{1}{2})$ est un maximum (local).

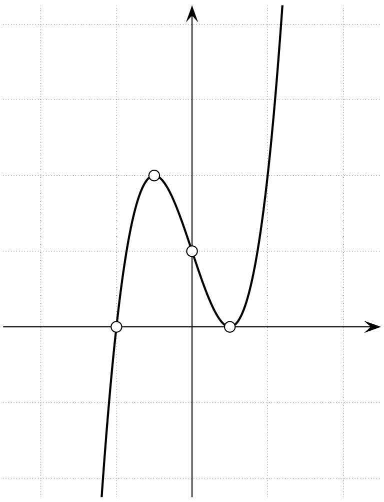
$f(\frac{1}{2}) = 4 \cdot (\frac{1}{2})^3 - 3 \cdot \frac{1}{2} + 1 = 0$
Le point $(\frac{1}{2}\,;0)$ est un minimum (local).

$f''(x) = (12\,x^2 - 3)' = 24\,x$

| | | $0$ | |
|---|---|---|---|
| $24\,x$ | $-$ | $0$ | $+$ |
| $f''$ | $-$ | $0$ | $+$ |
| $f$ | $\frown$ | infl | $\smile$ |

$f(0) = 4 \cdot 0^3 - 3 \cdot 0 + 1 = 1$

Le point $(0\,;1)$ est un point d'inflexion.



9) $f(x) = (x-1)^3\,(x+1)$

| | | $-1$ | | $1$ | |
|---|---|---|---|---|---|
| $(x-1)^3$ | $-$ | | $-$ | $0$ | $+$ |
| $x+1$ | $-$ | $0$ | $+$ | | $+$ |
| $f$ | $+$ | $0$ | $-$ | $0$ | $+$ |

$$f'(x) = \big((x-1)^3\,(x+1)\big)' = \big((x-1)^3\big)'\,(x+1) + (x-1)^3\,(x+1)'$$
$$= 3\,(x-1)^2\,\underbrace{(x-1)'}_{1}\,(x+1) + (x-1)^3 \cdot 1 = (x-1)^2\,\big(3\,(x+1) + (x-1)\big)$$
$$= (x-1)^2\,(4\,x+2) = 2\,(x-1)^2\,(2\,x+1)$$

| | | $-\frac{1}{2}$ | | $1$ | |
|---|---|---|---|---|---|
| $2$ | $+$ | | $+$ | | $+$ |
| $(x-1)^2$ | $+$ | | $+$ | $0$ | $+$ |
| $2\,x+1$ | $-$ | $0$ | $+$ | | $+$ |
| $f'$ | $-$ | $0$ | $+$ | $0$ | $+$ |
| $f$ | $\searrow$ | min | $\nearrow$ | replat | $\nearrow$ |

$f(-\frac{1}{2}) = (-\frac{1}{2} - 1)^3\,(-\frac{1}{2} + 1) = -\frac{27}{16}$

Le point $(-\frac{1}{2}\,;-\frac{27}{16})$ est un minimum (absolu).

$f(1) = (1-1)^3\,(1+1) = 0$

Le point $(0\,;0)$ est un replat.

$$f''(x) = 2\,\big((x-1)^2\,(2\,x+1)\big)' = 2\,\Big(\big((x-1)^2\big)'\,(2\,x+1) + (x-1)^2\,(2\,x+1)'\Big)$$

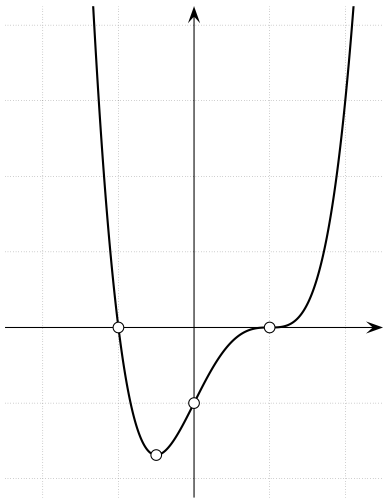
$$= 2\left(2\,(x-1)\,\underbrace{(x-1)'}_{1}\,(2\,x+1) + (x-1)^2 \cdot 2\right)$$

$$= 4\,(x-1)\,\big((2\,x+1)+(x-1)\big) = 4\,(x-1)\,(3\,x) = 12\,x\,(x-1)$$

| | | 0 | | 1 | |
|---|---|---|---|---|---|
| $12\,x$ | $-$ | $0$ | $+$ | | $+$ |
| $x-1$ | $-$ | | $-$ | $0$ | $+$ |
| $f''$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f$ | $\smile$ | infl | $\frown$ | infl | $\smile$ |

$f(0) = (0-1)^3\,(0+1) = -1$
Le point $(0\,;-1)$ est un point d'inflexion.

$f(1) = (1-1)^3\,(1+1) = 0$
Le point $(1\,;0)$ est un point d'inflexion.



10) $f(x) = \frac{1}{8}\,(x^4 - 6\,x^3 + 12\,x^2) = \frac{1}{8}\,x^2\,(x^2 - 6\,x + 12)$
$x^2 - 6\,x + 12$ n'admet aucun zéro, car $\Delta = (-6)^2 - 4 \cdot 1 \cdot 12 = -12 < 0$

| | | 0 | |
|---|---|---|---|
| $\frac{1}{8}$ | $+$ | | $+$ |
| $x^2$ | $+$ | $0$ | $+$ |
| $x^2 - 6\,x + 12$ | $+$ | | $+$ |
| $f$ | $+$ | $0$ | $+$ |

$f'(x) = \frac{1}{8}\,(x^4 - 6\,x^3 + 12\,x^2)' = \frac{1}{8}\,(4\,x^3 - 18\,x^2 + 24\,x) = \frac{1}{4}\,x\,(2\,x^2 - 9\,x + 12)$
$2\,x^2 - 9\,x + 12$ n'admet aucun zéro, vu que $\Delta = (-9)^2 - 4 \cdot 2 \cdot 12 = -15 < 0$

| | | 0 | |
|---|---|---|---|
| $\frac{1}{4}$ | $+$ | | $+$ |
| $x$ | $-$ | $0$ | $+$ |
| $2\,x^2 - 9\,x + 12$ | $+$ | | $+$ |
| $f'$ | $-$ | $0$ | $+$ |
| $f$ | $\searrow$ | min | $\nearrow$ |

$f(0) = \frac{1}{8}\,(0^4 - 6 \cdot 0^3 + 12 \cdot 0^2) = 0$
Le point $(0\,;0)$ est un minimum (absolu).

$$f''(x) = \tfrac{1}{4}\,(2\,x^3 - 9\,x^2 + 12\,x)' = \tfrac{1}{4}\,(6\,x^2 - 18\,x + 12) = \tfrac{3}{2}\,(x^2 - 3\,x + 2)$$
$$= \tfrac{3}{2}\,(x-1)\,(x-2)$$

| | | 1 | | 2 | |
|---|---|---|---|---|---|
| $\frac{3}{2}$ | $+$ | | $+$ | | $+$ |
| $x-1$ | $-$ | 0 | $+$ | | $+$ |
| $x-2$ | $-$ | | $-$ | 0 | $+$ |
| $f''$ | $+$ | 0 | $-$ | 0 | $+$ |
| $f$ | $\smile$ | infl | $\frown$ | infl | $\smile$ |

$f(1) = \frac{1}{8}\,(1^4 - 6 \cdot 1^3 + 12 \cdot 1^2) = \frac{7}{8}$
Le point $(1\,;\frac{7}{8})$ est un point d'inflexion.

$f(2) = \frac{1}{8}\,(2^4 - 6 \cdot 2^3 + 12 \cdot 2^2) = 2$
Le point $(2\,;2)$ est un point d'inflexion.