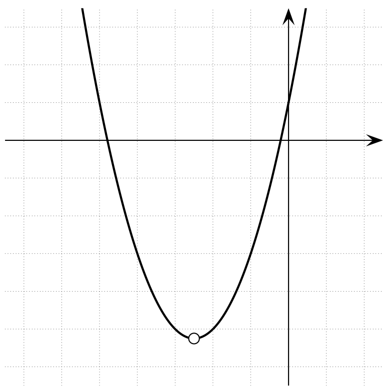
6.5 1) $f'(x) = (x^2 + 5\,x + 1)' = 2\,x + 5$

x		$-\frac{5}{2}$	
$2\,x+5$	$-$	0	$+$
f'	$-$	0	$+$
f	$\searrow$	min	$\nearrow$

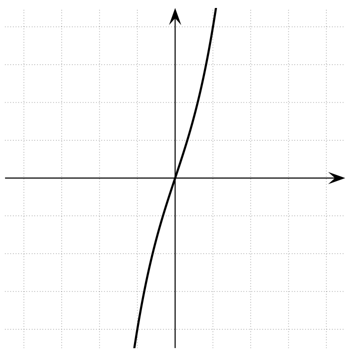
$f(-\frac{5}{2}) = (-\frac{5}{2})^2 + 5 \cdot (-\frac{5}{2}) + 1 = -\frac{21}{4}$

Le point $(-\frac{5}{2}\,; -\frac{21}{4})$ est un minimum absolu.



2) $f'(x) = (x^3 + 3\,x)' = 3\,x^2 + 3 = 3\,(x^2 + 1)$

3	$+$
x^2+1	$+$
f'	$+$
f	$\nearrow$



3) $f'(x) = (\frac{1}{3}\,x^3 + \frac{5}{2}\,x^2 + 6\,x + 1)' = x^2 + 5\,x + 6 = (x+3)\,(x+2)$

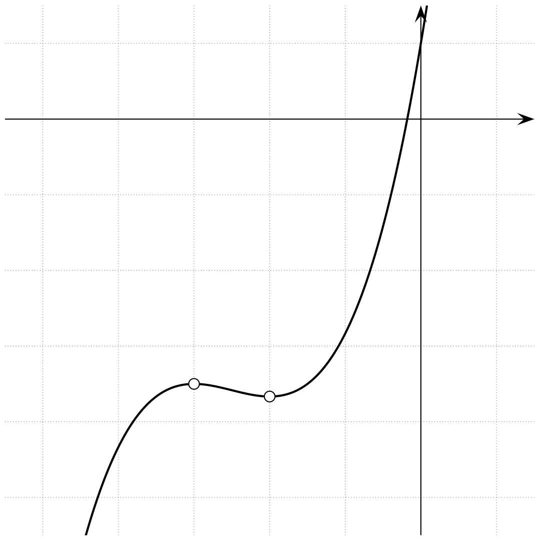
x		-3		-2	
$x+3$	$-$	0	$+$		$+$
$x+2$	$-$		$-$	0	$+$
f'	$+$	0	$-$	0	$+$
f	$\nearrow$	max	$\searrow$	min	$\nearrow$

$f(-3) = \frac{1}{3}\,(-3)^3 + \frac{5}{2}\,(-3)^2 + 6\,(-3) + 1 = -\frac{7}{2}$

Le point $(-3\,; -\frac{7}{2})$ est un maximum local.

$f(-2) = \frac{1}{3}\,(-2)^3 + \frac{5}{2}\,(-2)^2 + 6\,(-2) + 1 = -\frac{11}{3}$

Le point $(-2\,;-\frac{11}{3})$ est un minimum local.



4) $f'(x) = (2\,x^4 - 9\,x^2)' = 8\,x^3 - 18\,x = 2\,x\,(4\,x^2 - 9) = 2\,x\,(2\,x+3)\,(2\,x-3)$

		$-\frac{3}{2}$		0		$\frac{3}{2}$	
$2\,x$	$-$		$-$	0	$+$		$+$
$2\,x+3$	$-$	0	$+$		$+$		$+$
$2\,x-3$	$-$		$-$		$-$	0	$+$
f'	$-$	0	$+$	0	$-$	0	$+$
f	$\searrow$	min	$\nearrow$	max	$\searrow$	min	$\nearrow$

$f(-\frac{3}{2}) = 2\,(-\frac{3}{2})^4 - 9\,(-\frac{3}{2})^2 = -\frac{81}{8}$

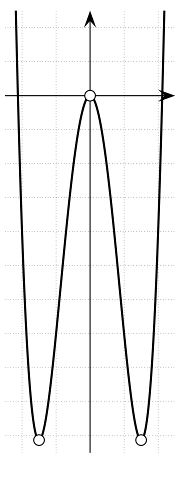
Le point $(-\frac{3}{2}\,;-\frac{81}{8})$ est un minimum local.

$f(0) = 2 \cdot 0^4 - 9 \cdot 0^2 = 0$

Le point $(0\,;0)$ est un maximum local.

$f(\frac{3}{2}) = 2\,(\frac{3}{2})^4 - 9\,(\frac{3}{2})^2 = -\frac{81}{8}$

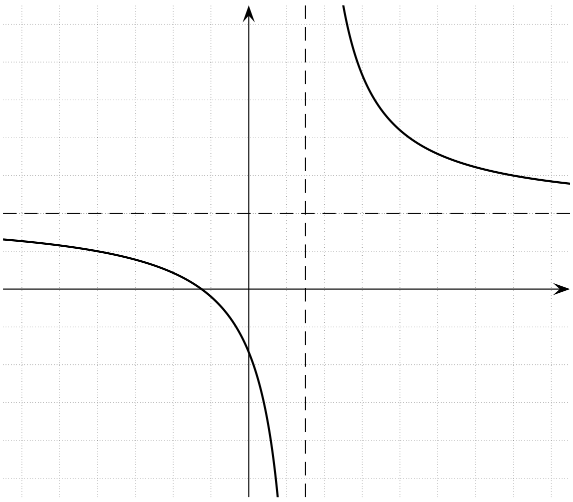
Le point $(\frac{3}{2}\,;-\frac{81}{8})$ est un minimum local.



5) $f'(x) = \left(\dfrac{4\,x+5}{2\,x-3}\right)' = \dfrac{(4\,x+5)'\,(2\,x-3)-(4\,x+5)\,(2\,x-3)'}{(2\,x-3)^2}$

$= \dfrac{4\,(2\,x-3)-2\,(4\,x+5)}{(2\,x-3)^2} = \dfrac{-22}{(2\,x-3)^2}$

		$\frac{3}{2}$	
-22	$-$	‖	$-$
$(2\,x-3)^2$	$+$	‖	$+$
f'	$-$	‖	$-$
f	↘	‖	↘



6) $f'(x) = \big((x-1)^5\,(2\,x+1)^4\big)' = \big((x-1)^5\big)'\,(2\,x+1)^4 + (x-1)^5\,\big((2\,x+1)^4\big)'$

$= 5\,(x-1)^4\,\underbrace{(x-1)'}_{1}\,(2\,x+1)^4 + (x-1)^5\,4\,(2\,x+1)^3\,\underbrace{(2\,x+1)'}_{2}$

$= 5\,(x-1)^4\,(2\,x+1)^4 + 8\,(x-1)^5\,(2\,x+1)^3$

$= (x-1)^4\,(2\,x+1)^3\,\big(5\,(2\,x+1)+8\,(x-1)\big)$

$= (x-1)^4\,(2\,x+1)^3\,\underbrace{(18\,x-3)}_{3\,(6\,x-1)}$

$= 3\,(x-1)^4\,(2\,x+1)^3\,(6\,x-1)$

		$-\frac{1}{2}$		$\frac{1}{6}$		1	
3	$+$	|	$+$	|	$+$	|	$+$
$(x-1)^4$	$+$	|	$+$	|	$+$	0	$+$
$(2\,x+1)^3$	$-$	0	$+$	|	$+$	|	$+$
$6\,x-1$	$-$	|	$-$	0	$+$	|	$+$
f'	$+$	0	$-$	0	$+$	0	$+$
f	↗	max	↘	min	↗	replat	↗

$f(-\frac{1}{2}) = (-\frac{1}{2}-1)^5\,\big(2\,(-\frac{1}{2})+1\big)^4 = (-\frac{3}{2})^5\,(0)^4 = -\frac{243}{32}\cdot 0 = 0$

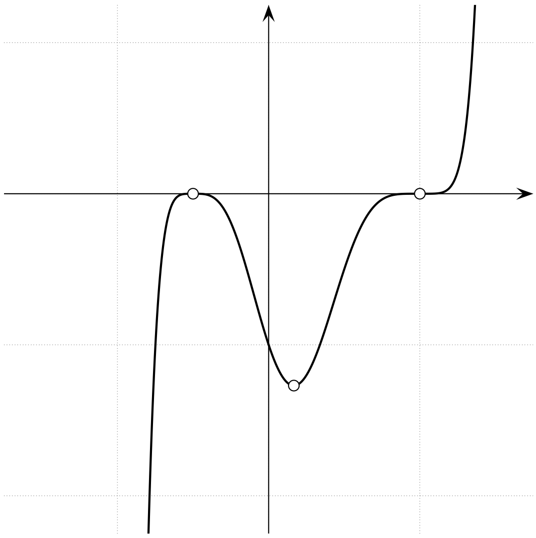
Le point $(-\frac{1}{2}\,;0)$ est un maximum local.

$f(\frac{1}{6}) = (\frac{1}{6} - 1)^5 \, (2 \cdot \frac{1}{6} + 1)^4 = (-\frac{5}{6})^5 \, (\frac{4}{3})^4 = -\frac{3125}{7776} \cdot \frac{256}{81} = -\frac{25000}{19683}$

Le point $(\frac{1}{6}\,; -\frac{25000}{19683})$ est un minimum local.

$f(1) = (1 - 1)^5 \, (2 \cdot 1 + 1)^4 = 0^5 \cdot 3^4 = 0$

Le point $(1\,;0)$ est un replat.



7) $f'(x) = (x^5 - 5\,x^4 + 5\,x^3 + 1)' = 5\,x^4 - 20\,x^3 + 15\,x^2 = 5\,x^2\,(x^2 - 4\,x + 3)$
$= 5\,x^2\,(x - 1)\,(x - 3)$

		0		1		3	
5	$+$		$+$		$+$		$+$
x^2	$+$	0	$+$		$+$		$+$
$x-1$	$-$		$-$	0	$+$		$+$
$x-3$	$-$		$-$		$-$	0	$+$
f'	$+$	0	$+$	0	$-$	0	$+$
f	$\nearrow$	replat	$\nearrow$	max	$\searrow$	min	$\nearrow$

$f(0) = 0^5 - 5 \cdot 0^4 + 5 \cdot 0^3 + 1 = 1$

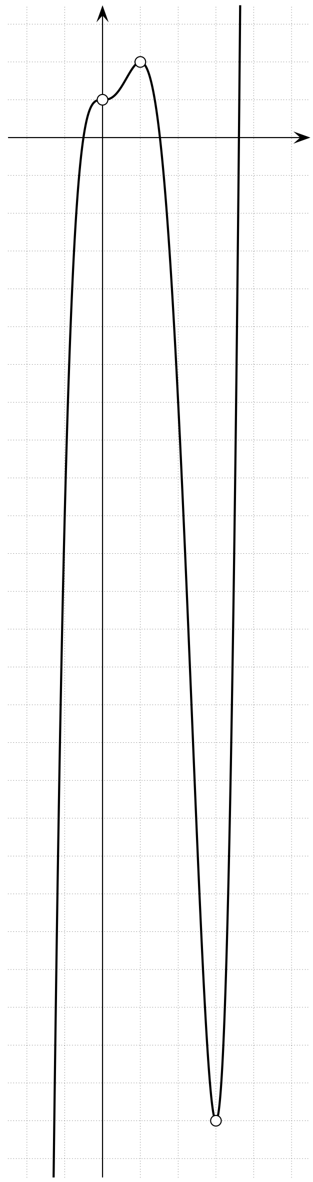
Le point $(0\,;1)$ est un replat.

$f(1) = 1^5 - 5 \cdot 1^4 + 5 \cdot 1^3 + 1 = 2$

Le point $(1\,;2)$ est un maximum local.

$f(3) = 3^5 - 5 \cdot 3^4 + 5 \cdot 3^3 + 1 = -26$

Le point $(3\,;-26)$ est un minimum local.



8) $f'(x) = (x^3 + \dfrac{3}{x})' = (x^3 + 3\,x^{-1})' = 3\,x^2 - 3\,x^{-2} = 3\,x^2 - \dfrac{3}{x^2} = \dfrac{3\,x^4 - 3}{x^2}$

$$= \frac{3\,(x^4-1)}{x^2} = \frac{3\,(x^2-1)\,(x^2+1)}{x^2} = \frac{3\,(x-1)\,(x+1)\,(x^2+1)}{x^2}$$

		-1		0		1	
3	$+$		$+$	‖	$+$		$+$
$x-1$	$-$		$-$	‖	$-$	0	$+$
$x+1$	$-$	0	$+$	‖	$+$		$+$
x^2+1	$+$		$+$	‖	$+$		$+$
x^2	$+$		$+$	‖	$+$		$+$
f'	$+$	0	$-$	‖	$-$	0	$+$
f	$\nearrow$	max	$\searrow$	‖	$\searrow$	min	$\nearrow$

$f(-1) = (-1)^3 + \frac{3}{-1} = -4$
Le point $(-1\,;-4)$ est un maximum local.

$f(1) = 1^3 + \frac{3}{1} = 4$
Le point $(1\,;4)$ est un minimum local.