5.15 Soient $p \in \mathbb{Z}$ et $q \in \mathbb{Z} - \{0\}$.

$\big((x^{\frac{p}{q}})^q\big)' = (x^{\frac{p}{q} \cdot q})' = (x^p)' = p\, x^{p-1}$

Posons $f(x) = x^{\frac{p}{q}}$.

$\big((x^{\frac{p}{q}})^q\big)' = \big(f^q(x)\big)' = q\, f^{q-1}(x) \cdot f'(x) = q\,(x^{\frac{p}{q}})^{q-1} \cdot (x^{\frac{p}{q}})' = q\, x^{\frac{p\,(q-1)}{q}} \cdot (x^{\frac{p}{q}})'$

L'égalité $p\, x^{p-1} = q\, x^{\frac{p\,(q-1)}{q}} \cdot (x^{\frac{p}{q}})'$ implique

$(x^{\frac{p}{q}})' = \frac{p}{q}\, x^{(p-1) - \frac{p\,(q-1)}{q}} = \frac{p}{q}\, x^{\frac{(p-1)\,q - p\,(q-1)}{q}} = \frac{p}{q}\, x^{\frac{p\,q - q - p\,q + p}{q}} = \frac{p}{q}\, x^{\frac{p-q}{q}} = \frac{p}{q}\, x^{\frac{p}{q} - \frac{q}{q}}$

$\quad = \frac{p}{q}\, x^{\frac{p}{q} - 1}$