5.19 1) $f'(x) = \left(\dfrac{(x-1)^3}{(x+1)^2}\right)'$

$$= \frac{\left((x-1)^3\right)'(x+1)^2 - (x-1)^3\left((x+1)^2\right)'}{\left((x+1)^2\right)^2}$$

$$= \frac{3(x-1)^2\overbrace{(x-1)'}^{1}(x+1)^2 - (x-1)^3 2(x+1)\overbrace{(x+1)'}^{1}}{(x+1)^4}$$

$$= \frac{3(x-1)^2(x+1)^2 - 2(x-1)^3(x+1)}{(x+1)^4}$$

$$= \frac{(x-1)^2(x+1)\left(3(x+1) - 2(x-1)\right)}{(x+1)^4}$$

$$= \frac{(x-1)^2(3x+3-2x+2)}{(x+1)^3}$$

$$= \frac{(x-1)^2(x+5)}{(x+1)^3}$$

2) $f'(x) = \left(\dfrac{(3x-1)^3}{(2x+3)^2}\right)'$

$$= \frac{\left((3x-1)^3\right)'(2x+3)^2 - (3x-1)^3\left((2x+3)^2\right)'}{\left((2x+3)^2\right)^2}$$

$$= \frac{3(3x-1)^2\overbrace{(3x-1)'}^{3}(2x+3)^2 - (3x-1)^3 2(2x+3)\overbrace{(2x+3)'}^{2}}{(2x+3)^4}$$

$$= \frac{9(3x-1)^2(2x+3)^2 - 4(3x-1)^3(2x+3)}{(2x+3)^4}$$

$$= \frac{(3x-1)^2(2x+3)\left(9(2x+3) - 4(3x-1)\right)}{(2x+3)^4}$$

$$= \frac{(3x-1)^2(18x+27-12x+4)}{(2x+3)^3}$$

$$= \frac{(3x-1)^2(6x+31)}{(2x+3)^3}$$

3) $f'(x) = \left(\dfrac{(x-4)(3x-7)}{x^2-4x+2}\right)' =$

$$\frac{\left((x-4)(3x-7)\right)'(x^2-4x+2) - (x-4)(3x-7)(x^2-4x+2)'}{(x^2-4x+2)^2} =$$

$$\frac{\overbrace{\left((x-4)'\right)}^{1}(3x-7)+(x-4)\overbrace{(3x-7)'}^{3}(x^2-4x+2)-(x-4)(3x-7)(2x-4)}{(x^2-4x+2)^2}=$$

$$\frac{(3x-7+3x-12)(x^2-4x+2)-(3x^2-7x-12x+28)(2x-4)}{(x^2-4x+2)^2}=$$

$$\frac{(6x-19)(x^2-4x+2)-(3x^2-19x+28)(2x-4)}{(x^2-4x+2)^2}=$$

$$\frac{6x^3-24x^2+12x-19x^2+76x-38-6x^3+12x^2+38x^2-76x-56x+112}{(x^2-4x+2)^2}=$$

$$\frac{7x^2-44x+74}{(x^2-4x+2)^2}$$

4) $f'(x)=\left(\dfrac{(x-5)(3-2x)}{4x+2}\right)'$

$$=\frac{\left((x-5)(3-2x)\right)'(4x+2)-(x-5)(3-2x)(4x+2)'}{(4x+2)^2}$$

$$=\frac{\overbrace{\left((x-5)'\right)}^{1}(3-2x)+(x-5)\overbrace{(3-2x)'}^{-2})(4x+2)-(x-5)(3-2x)4}{\left(2(2x+1)\right)^2}$$

$$=\frac{(3-2x-2x+10)(4x+2)-(x-5)(3-2x)4}{4(2x+1)^2}$$

$$=\frac{(-4x+13)(4x+2)-(x-5)(12-8x)}{4(2x+1)^2}$$

$$=\frac{-16x^2-8x+52x+26-12x+8x^2+60-40x}{4(2x+1)^2}$$

$$=\frac{-8x^2-8x+86}{4(2x+1)^2}$$

$$=\frac{2(-4x^2-4x+43)}{4(2x+1)^2}$$

$$=\frac{-4x^2-4x+43}{2(2x+1)^2}$$

5) $f'(x) = \left(3\,x - 2 - \dfrac{1}{3\,x-2}\right)'$

$= (3\,x-2)' + \left(-\dfrac{1}{3\,x-2}\right)'$

$= 3 - \dfrac{-\overbrace{(3\,x-2)'}^{3}}{(3\,x-2)^2}$

$= 3 + \dfrac{3}{(3\,x-2)^2}$

$= \dfrac{3\,(3\,x-2)^2+3}{(3\,x-2)^2}$

$= \dfrac{3\left((3\,x-2)^2+1\right)}{(3\,x-2)^2}$

$= \dfrac{3\,(9\,x^2-12\,x+4+1)}{(3\,x-2)^2}$

$= \dfrac{3\,(9\,x^2-12\,x+5)}{(3\,x-2)^2}$

6) $f'(x) = \left(\dfrac{1}{(x-2)^2} + \dfrac{1}{(x+2)^2}\right)'$

$= \left(\dfrac{1}{(x-2)^2}\right)' + \left(\dfrac{1}{(x+2)^2}\right)'$

$= \dfrac{-\left((x-2)^2\right)'}{\left((x-2)^2\right)^2} + \dfrac{-\left((x+2)^2\right)'}{\left((x+2)^2\right)^2}$

$= \dfrac{-2\,(x-2)\,\overbrace{(x-2)'}^{1}}{(x-2)^4} + \dfrac{-2\,(x+2)\,\overbrace{(x+2)'}^{1}}{(x+2)^4}$

$= -\dfrac{2}{(x-2)^3} - \dfrac{2}{(x+2)^3}$

$= \dfrac{-2\,(x+2)^3 - 2\,(x-2)^3}{(x-2)^3\,(x+2)^3}$

$= \dfrac{-2\left((x+2)^3 + (x-2)^3\right)}{(x-2)^3\,(x+2)^3}$

$= \dfrac{-2\,(x^3+6\,x^2+12\,x+8+x^3-6\,x^2+12\,x-8)}{(x-2)^3\,(x+2)^3}$

$= \dfrac{-2\,(2\,x^3+24\,x)}{(x-2)^3\,(x+2)^3}$

$$= \frac{-2 \cdot 2\, x\, (x^2 + 12)}{(x-2)^3\, (x+2)^3}$$
$$= \frac{-4\, x(x^2 + 12)}{(x-2)^3\, (x+2)^3}$$