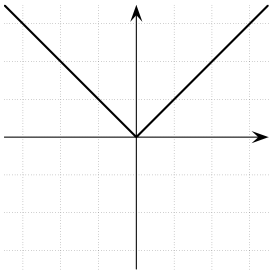
5.2 1) 

2) Soit $x > 0$.

Pour peu que h soit suffisamment proche de 0, on a $x + h > 0$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{|x+h| - |x|}{h} = \lim_{h \to 0} \frac{x+h-x}{h}$$
$$= \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

3) Soit $x < 0$.

Pour autant que h soit suffisamment proche de 0, on a $x + h < 0$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{|x+h| - |x|}{h} = \lim_{h \to 0} \frac{-(x+h) - (-x)}{h}$$
$$= \lim_{h \to 0} \frac{-h}{h} = \lim_{h \to 0} -1 = -1$$

4) (a) $$\lim_{\substack{h \to 0 \\ h > 0}} \frac{f(0+h) - f(0)}{h} = \lim_{\substack{h \to 0 \\ h > 0}} \frac{|0+h| - |0|}{h} = \lim_{\substack{h \to 0 \\ h > 0}} \frac{|h|}{h} = \lim_{\substack{h \to 0 \\ h > 0}} \frac{h}{h} = \lim_{\substack{h \to 0 \\ h > 0}} 1 = 1$$

(b) $$\lim_{\substack{h \to 0 \\ h < 0}} \frac{f(0+h) - f(0)}{h} = \lim_{\substack{h \to 0 \\ h < 0}} \frac{|0+h| - |0|}{h} = \lim_{\substack{h \to 0 \\ h < 0}} \frac{|h|}{h} = \lim_{\substack{h \to 0 \\ h < 0}} \frac{-h}{h} = \lim_{\substack{h \to 0 \\ h < 0}} -1 = -1$$

(c) Étant donné que $\displaystyle\lim_{\substack{h \to 0 \\ h > 0}} \frac{f(0+h) - f(0)}{h} \neq \lim_{\substack{h \to 0 \\ h < 0}} \frac{f(0+h) - f(0)}{h}$,

la limite $\displaystyle\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$ n'est pas définie, de sorte que la fonction $f(x) = |x|$ n'est pas dérivable en 0.