5.26 On rappelle que l'angle φ entre deux droites de pentes respectives m_1 et m_2 s'obtient par la formule $\tan(\varphi) = \left|\dfrac{m_2 - m_1}{1 + m_1\, m_2}\right|$.

1) Posons $f(x) = x^2$ et $g(x) = x^3$.
On a $f'(x) = 2\,x$ et $g'(x) = 3\,x^2$.

Calculons l'intersection entre les deux courbes :
$f(x) = g(x)$
$x^2 = x^3$
$0 = x^3 - x^2 = x^2\,(x-1)$
$x = 0$ ou $x = 1$

(a) $x = 0$ implique $y = f(0) = g(0) = 0$
$m_1 = f'(0) = 2 \cdot 0 = 0$
$m_2 = g'(0) = 3 \cdot 0^2 = 0$
$\tan(\varphi) = \left|\dfrac{0-0}{1+0\cdot 0}\right| = 0$ entraîne $\varphi = 0^\circ$

(b) $x = 1$ donne $y = f(1) = g(1) = 1$
$m_1 = f'(1) = 2 \cdot 1 = 2$
$m_2 = g'(1) = 3 \cdot 1^2 = 3$
$\tan(\varphi) = \left|\dfrac{3-2}{1+2\cdot 3}\right| = \dfrac{1}{7}$ fournit $\varphi \approx 8,13^\circ$

2) Posons $f(x) = x^2$ et $g(x) = \frac{1}{4}\,x^2 + 3$
Alors $f'(x) = 2\,x$ et $g'(x) = \frac{1}{2}\,x$

Calculons l'intersection entre les deux courbes :
$f(x) = g(x)$
$x^2 = \frac{1}{4}\,x^2 + 3$
$\frac{3}{4}\,x^2 - 3 = 0$
$x^2 - 4 = (x+2)\,(x-2) = 0$
$x = -2$ ou $x = 2$

(a) $x = -2$ implique $y = f(-2) = g(-2) = 4$
$m_1 = f'(-2) = 2 \cdot (-2) = -4$
$m_2 = g'(-2) = \frac{1}{2}\,(-2) = -1$
$\tan(\varphi) = \left|\dfrac{-1-(-4)}{1+(-1)\cdot(-4)}\right| = \dfrac{3}{5}$ délivre $\varphi \approx 30,96^\circ$

(b) $x = 2$ entraîne $y = f(2) = g(2) = 4$
$m_1 = f'(2) = 2 \cdot 2 = 4$
$m_2 = g'(2) = \frac{1}{2} \cdot 2 = 1$
$\tan(\varphi) = \left|\dfrac{1-4}{1+1\cdot 4}\right| = \dfrac{3}{5}$ implique $\varphi \approx 30,96^\circ$

3) Posons $f(x) = \frac{1}{4}\,x^2$ et $g(x) = -x^2 + 10\,x - 15$

Alors $f'(x) = \frac{1}{2}\,x$ et $g'(x) = -2\,x + 10$

Calculons l'intersection entre les deux courbes :

$f(x) = g(x)$

$\frac{1}{4}\,x^2 = -x^2 + 10\,x - 15$

$\frac{5}{4}\,x^2 - 10\,x + 15 = 0$

$x^2 - 8\,x + 12 = (x-2)\,(x-6) = 0$

$x = 2$ ou $x = 6$

(a) $x = 2$ donne $y = f(2) = g(2) = 1$

$m_1 = f'(2) = \frac{1}{2} \cdot 2 = 1$

$m_2 = g'(2) = -2 \cdot 2 + 10 = 6$

$\tan(\varphi) = \left|\dfrac{6-1}{1+1\cdot 6}\right| = \dfrac{5}{7}$ délivre $\varphi \approx 35,54^\circ$

(b) $x = 6$ implique $y = f(6) = g(6) = 9$

$m_1 = f'(6) = \frac{1}{2} \cdot 6 = 3$

$m_2 = g'(6) = -2 \cdot 6 + 10 = -2$

$\tan(\varphi) = \left|\dfrac{-2-3}{1+3\cdot(-2)}\right| = 1$ fournit $\varphi = 45^\circ$

4) Posons $f(x) = x^3 - 4\,x$ et $g(x) = x^3 - 2\,x^2$

Alors $f'(x) = 3\,x^2 - 4$ et $g'(x) = 3\,x^2 - 4\,x$

Calculons l'intersection entre les deux courbes :

$f(x) = g(x)$

$x^3 - 4\,x = x^3 - 2\,x^2$

$2\,x^2 - 4\,x = 2\,x\,(x-2) = 0$

$x = 0$ ou $x = 2$

(a) $x = 0$ implique $y = f(0) = g(0) = 0$

$m_1 = f'(0) = 3 \cdot 0^2 - 4 = -4$

$m_2 = g'(0) = 3 \cdot 0^2 - 4 \cdot 0 = 0$

$\tan(\varphi) = \left|\dfrac{0-(-4)}{1+(-4)\cdot 0}\right| = 4$ conduit à $\varphi \approx 75,96^\circ$

(b) $x = 2$ fournit $y = f(2) = g(2) = 0$

$m_1 = f'(2) = 3 \cdot 2^2 - 4 = 8$

$m_2 = g'(2) = 3 \cdot 2^2 - 4 \cdot 2 = 4$

$\tan(\varphi) = \left|\dfrac{4-8}{1+8\cdot 4}\right| = \dfrac{4}{33}$ donne $\varphi \approx 6,91^\circ$