7.17 1) $\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)=\displaystyle\sum_{k=0}^{+\infty}(-1)^k\,\frac{\left(\frac{x}{2}\right)^{2k+1}}{(2\,k+1)!}+\sum_{k=0}^{+\infty}(-1)^k\,\frac{\left(\frac{x}{2}\right)^{2k}}{(2\,k)!}$

$$=\sum_{k=0}^{+\infty}(-1)^k\,\frac{\left(\frac{x}{2}\right)^{2k+1}}{(2\,k+1)!}+(-1)^k\,\frac{\left(\frac{x}{2}\right)^{2k}}{(2\,k)!}$$

$$=\sum_{k=0}^{+\infty}(-1)^k\,\frac{\left(\frac{x}{2}\right)^{2k}}{(2\,k)!}+(-1)^k\,\frac{\left(\frac{x}{2}\right)^{2k+1}}{(2\,k+1)!}$$

$$=\sum_{k=0}^{+\infty}(-1)^k\,\frac{1}{2^{2k}\,(2\,k)!}\,x^{2k}+(-1)^k\,\frac{1}{2^{2k+1}\,(2\,k+1)!}\,x^{2k+1}$$

$$=1+\frac{1}{2}\,x-\frac{1}{2^2\,2!}\,x^2-\frac{1}{2^3\,3!}\,x^3+\frac{1}{2^4\,4!}\,x^4+\frac{1}{2^5\,5!}\,x^5+\ldots$$

2) $\cos^2(x)=\dfrac{1+\cos(2\,x)}{2}=\frac{1}{2}+\frac{1}{2}\,\cos(2\,x)=\frac{1}{2}+\frac{1}{2}\displaystyle\sum_{k=0}^{+\infty}(-1)^k\,\frac{(2\,x)^{2k}}{(2\,k)!}$

$$=\tfrac{1}{2}+\tfrac{1}{2}\sum_{k=0}^{+\infty}(-1)^k\,\frac{2^{2k}\,x^{2k}}{(2\,k)!}=\tfrac{1}{2}+\sum_{k=0}^{+\infty}(-1)^k\,\tfrac{1}{2}\cdot\frac{2^{2k}}{(2\,k)!}\,x^{2k}$$

$$=\tfrac{1}{2}+\sum_{k=0}^{+\infty}(-1)^k\,\frac{2^{2k-1}}{(2\,k)!}\,x^{2k}$$

$$=\underbrace{\tfrac{1}{2}+\tfrac{1}{2}}_{=1}-\frac{2}{2!}\,x^2+\frac{2^3}{4!}\,x^3-\frac{2^5}{6!}\,x^6+\ldots+(-1)^k\,\frac{2^{2k-1}}{(2k)!}\,x^{2k}+\ldots$$