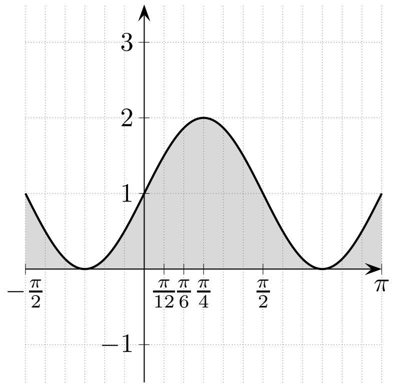
Chamblandes 2016 — Problème 3

A. a) Voici le graphe de f dans l'intervalle $[-\frac{\pi}{2}\,;\pi]$:



b) $\displaystyle\int_{-\frac{\pi}{2}}^{\pi}\Big(1+\sin(2\,x)\Big)\,dx=\int_{-\frac{\pi}{2}}^{\pi}\Big(1+\tfrac{1}{2}\,\sin(2\,x)\cdot 2\Big)\,dx=x-\tfrac{1}{2}\,\cos(2\,x)\,\Big|_{-\frac{\pi}{2}}^{\pi}=$

$\Big(\pi-\frac{1}{2}\,\cos(2\,\pi)\Big)-\Big(-\frac{\pi}{2}-\frac{1}{2}\,\cos(2\cdot(-\frac{\pi}{2}))\Big)=\Big(\pi-\frac{1}{2}\cdot 1\Big)-\Big(-\frac{\pi}{2}-\frac{1}{2}\cdot(-1)\Big)=$

$\pi-\frac{1}{2}+\frac{\pi}{2}-\frac{1}{2}=\frac{3\,\pi}{2}-1$

B. $\displaystyle\pi\int_0^7\Big(\sqrt[3]{x+1}\Big)^2\,dx=\pi\int_0^7\Big((x+1)^{\frac{1}{3}}\Big)^2\,dx=\pi\int_0^7(x+1)^{\frac{2}{3}}\,dx=\pi\int_0^7(x+1)^{\frac{2}{3}}\cdot 1\,dx=$

$\displaystyle\pi\left(\frac{1}{\frac{2}{3}+1}\,(x+1)^{\frac{2}{3}+1}\,\Bigg|_0^7\right)=\pi\left(\tfrac{3}{5}\,(x+1)^{\frac{5}{3}}\,\Bigg|_0^7\right)=\tfrac{3\,\pi}{5}\,\sqrt[3]{(x+1)^5}\,\Bigg|_0^7=$

$\frac{3\,\pi}{5}\left(\sqrt[3]{(7+1)^5}-\sqrt[3]{(0+1)^5}\right)=\frac{3\,\pi}{5}\,(32-1)=\frac{93\,\pi}{5}$