9.15

1) $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = \dfrac{e^0 - 1}{0} = \dfrac{1-1}{0} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{x\to 0} \frac{(e^x-1)'}{(x)'} = \lim_{x\to 0} \frac{e^x}{1} = \lim_{x\to 0} e^x = e^0 = 1$$

2) $\lim\limits_{x\to 2} \dfrac{e^x - e^2}{x-2} = \lim\limits_{x\to 2} \dfrac{e^2 - e^2}{2-2} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 2} \frac{e^x - e^2}{x-2} = \lim_{x\to 2} \frac{(e^x - e^2)'}{(x-2)'} = \lim_{x\to 2} \frac{e^x - 0}{1-0} = \lim_{x\to 2} \frac{e^x}{1} = \lim_{x\to 2} e^x = e^2$$

3) $\lim\limits_{x\to 0} \dfrac{x\,e^x}{1-e^x} = \dfrac{0\cdot e^0}{1-e^0} = \dfrac{0\cdot 1}{1-1} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 0} \frac{x\,e^x}{1-e^x} = \lim_{x\to 0} \frac{(x\,e^x)'}{(1-e^x)'} = \lim_{x\to 0} \frac{(x)'\,e^x + x\,(e^x)'}{0-e^x} = \lim_{x\to 0} \frac{1\cdot e^x + x\,e^x}{-e^x}$$

$$= \lim_{x\to 0} \frac{e^x\,(1+x)}{-e^x} = \lim_{x\to 0} -(1+x) = -(1+0) = -1$$

4) $\lim\limits_{x\to -1} \dfrac{\ln(2+x)}{x+1} = \dfrac{\ln\big(2+(-1)\big)}{-1+1} = \dfrac{\ln(1)}{0} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to -1} \frac{\ln(2+x)}{x+1} = \lim_{x\to -1} \frac{\big(\ln(2+x)\big)'}{(x+1)'} = \lim_{x\to -1} \frac{\ln'(2+x)\,(2+x)'}{1}$$

$$= \lim_{x\to -1} \frac{1}{2+x}\cdot 1 = \lim_{x\to -1} \frac{1}{2+x} = \frac{1}{2+(-1)} = 1$$

5) $\lim\limits_{x\to e} \dfrac{\ln(x)-1}{x-e} = \dfrac{\ln(e)-1}{e-e} = \dfrac{1-1}{0} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to e} \frac{\ln(x)-1}{x-e} = \lim_{x\to e} \frac{\big(\ln(x)-1\big)'}{(x-e)'} = \lim_{x\to e} \frac{\frac{1}{x}-0}{1-0} = \lim_{x\to e} \frac{1}{x} = \frac{1}{e}$$

6) $\lim\limits_{x\to 1} \dfrac{x-1}{\ln(x)} = \dfrac{1-1}{\ln(1)} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 1} \frac{x-1}{\ln(x)} = \lim_{x\to 1} \frac{(x-1)'}{\big(\ln(x)\big)'} = \lim_{x\to 1} \frac{1}{\dfrac{1}{x}} = \lim_{x\to 1} x = 1$$

7) $\lim\limits_{x\to 0_+} \dfrac{\ln(x)}{x^2} = \dfrac{\ln(0_+)}{0_+^2} = \dfrac{-\infty}{0_+} = -\infty$

8) $\lim\limits_{x\to 2} \dfrac{\ln(x^2-3)}{x-2} = \dfrac{\ln(2^2-3)}{2-2} = \dfrac{\ln(1)}{0} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 2} \frac{\ln(x^2-3)}{x-2} = \lim_{x\to 2} \frac{\big(\ln(x^2-3)\big)'}{(x-2)'} = \lim_{x\to 2} \frac{\ln'(x^2-3)\,(x^2-3)'}{1}$$

$$= \lim_{x\to 2} \frac{1}{x^2-3}\cdot(2\,x) = \lim_{x\to 2} \frac{2\,x}{x^2-3} = \frac{2\cdot 2}{2^2-3} = \frac{4}{1} = 4$$

9) $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{x} = \dfrac{e^{2\cdot 0}-1}{0} = \dfrac{e^{0}-1}{0} = \dfrac{1-1}{0} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 0} \frac{e^{2x}-1}{x} = \lim_{x\to 0} \frac{(e^{2x}-1)'}{(x)'} = \lim_{x\to 0} \frac{e^{2x}\,(2\,x)'-0}{1} = \lim_{x\to 0} e^{2x}\cdot 2 = \lim_{x\to 0} 2\,e^{2x}$$
$$= 2\cdot e^{2\cdot 0} = 2\cdot e^{0} = 2\cdot 1 = 2$$

10) $\lim\limits_{x\to 0_-} x\,e^{\frac{1}{x}} = 0_-\cdot e^{\frac{1}{0_-}} = 0_-\cdot e^{-\infty} = 0_-\cdot 0_+ = 0_-$

11) $\lim\limits_{x\to 0_+} x\,e^{\frac{1}{x}} = 0_+\cdot e^{\frac{1}{0_+}} = 0_+\cdot e^{+\infty} = 0_+\cdot(+\infty)$: indéterminé

$$\lim_{x\to 0_+} x\,e^{\frac{1}{x}} = \lim_{x\to 0_+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} = \lim_{x\to 0_+} \frac{(e^{\frac{1}{x}})'}{(\frac{1}{x})'} = \lim_{x\to 0_+} \frac{e^{\frac{1}{x}}\,(\frac{1}{x})'}{(\frac{1}{x})'} = \lim_{x\to 0_+} e^{\frac{1}{x}}$$
$$= e^{\frac{1}{0_+}} = e^{+\infty} = +\infty$$

12) $\lim\limits_{x\to +\infty} x\,e^{\frac{1}{x}} = (+\infty)\cdot e^{\frac{1}{+\infty}} = (+\infty)\cdot e^{0_+} = (+\infty)\cdot 1_+ = +\infty$

13) $\lim\limits_{x\to -\infty} x\,e^{\frac{1}{x}} = (-\infty)\,e^{\frac{1}{-\infty}} = (-\infty)\,e^{0_-} = (-\infty)\cdot 1_- = -\infty$

14) $\lim\limits_{x\to 0} \dfrac{e^{x}-1}{x^2+2\,x} = \dfrac{e^{0}-1}{0^2+2\cdot 0} = \dfrac{1-1}{0+0} = \dfrac{0}{0}$: indéterminé

$$\lim_{x\to 0} \frac{e^{x}-1}{x^2+2\,x} = \lim_{x\to 0} \frac{(e^{x}-1)'}{(x^2+2\,x)'} = \lim_{x\to 0} \frac{e^{x}-0}{2\,x+2} = \lim_{x\to 0} \frac{e^{x}}{2\,x+2}$$
$$= \frac{e^{0}}{2\cdot 0+2} = \frac{1}{2}$$

15) $\lim\limits_{x\to +\infty} \dfrac{2\,x+3}{x\,\ln(x)} = \dfrac{2\cdot(+\infty)+3}{(+\infty)\cdot\ln(+\infty)} = \dfrac{+\infty+3}{(+\infty)\cdot(+\infty)} = \dfrac{+\infty}{+\infty}$: indéterminé

$$\lim_{x\to +\infty} \frac{2\,x+3}{x\,\ln(x)} = \lim_{x\to +\infty} \frac{(2\,x+3)'}{\big(x\,\ln(x)\big)'} = \lim_{x\to +\infty} \frac{2}{(x)'\,\ln(x)+x\,\big(\ln(x)\big)'}$$
$$= \lim_{x\to +\infty} \frac{2}{1\cdot\ln(x)+x\cdot\frac{1}{x}} = \lim_{x\to +\infty} \frac{2}{\ln(x)+1}$$
$$= \frac{2}{\ln(+\infty)+1} = \frac{2}{+\infty+1} = \frac{2}{+\infty} = 0$$

16) $\lim\limits_{x\to 0_+} x\,\ln\left(1+\dfrac{1}{x}\right) = 0_+\cdot\ln\Big(1+\dfrac{1}{0_+}\Big) = 0_+\cdot\ln\big(1+(+\infty)\big)$

$$= 0_+\cdot\ln(+\infty) = 0_+\cdot(+\infty) \text{ : indéterminé}$$

$$\begin{aligned}
\lim_{x\to 0_+} x\,\ln\left(1+\frac{1}{x}\right) &= \lim_{x\to 0_+} \frac{\ln\bigl(1+\frac{1}{x}\bigr)}{\frac{1}{x}} = \lim_{x\to 0_+} \frac{\Bigl(\ln\bigl(1+\frac{1}{x}\bigr)\Bigr)'}{\bigl(\frac{1}{x}\bigr)'} \\
&= \lim_{x\to 0_+} \frac{\ln'\bigl(1+\frac{1}{x}\bigr)\cdot\bigl(1+\frac{1}{x}\bigr)'}{\bigl(\frac{1}{x}\bigr)'} = \lim_{x\to 0_+} \frac{\dfrac{1}{1+\frac{1}{x}}\cdot\bigl(\frac{1}{x}\bigr)'}{\bigl(\frac{1}{x}\bigr)'} \\
&= \lim_{x\to 0_+} \frac{1}{1+\frac{1}{x}} = \frac{1}{1+\frac{1}{0_+}} = \frac{1}{1+(+\infty)} = \frac{1}{+\infty} = 0
\end{aligned}$$