8.11

1) (a) $f'(x) = \big(\sin(3\,x + \frac{\pi}{4})\big)' = \sin'(3\,x + \frac{\pi}{4})\,(3\,x + \frac{\pi}{4})' = \cos(3\,x + \frac{\pi}{4}) \cdot 3$
$= 3\,\cos(3\,x + \frac{\pi}{4})$

(b) $f''(x) = \big(3\,\cos(3\,x + \frac{\pi}{4})\big)' = 3\,\big(\cos(3\,x + \frac{\pi}{4})\big)'$
$= 3\,\cos'(3\,x + \frac{\pi}{4})\,(3\,x + \frac{\pi}{4})' = 3\,\big(-\sin(3\,x + \frac{\pi}{4})\big) \cdot 3$
$= -9\,\sin(3\,x + \frac{\pi}{4})$

2) (a) $f'(x) = \big(\cos(x) + \sin(x)\big)' = -\sin(x) + \cos(x)$

(b) $f''(x) = \big(-\sin(x) + \cos(x)\big)' = -\cos(x) - \sin(x)$

3) (a) $f'(x) = \big(\sin(x)\,\cos(x)\big)' = \sin'(x)\,\cos(x) + \sin(x)\,\cos'(x)$
$= \cos(x)\,\cos(x) + \sin(x)\,\big(-\sin(x)\big) = \cos^2(x) - \sin^2(x)$

(b) $f''(x) = \big(\cos^2(x) - \sin^2(x)\big)' = 2\,\cos(x)\,\cos'(x) - 2\,\sin(x)\,\sin'(x)$
$= 2\,\cos(x)\,\big(-\sin(x)\big) - 2\,\sin(x)\,\cos(x)$
$= -2\,\sin(x)\,\cos(x) - 2\,\sin(x)\,\cos(x) = -4\,\sin(x)\,\cos(x)$

4) (a) $f'(x) = \big(\cos(x) + \sin^2(x) - 1\big)' = -\sin(x) + 2\,\sin(x)\,\sin'(x)$
$= -\sin(x) + 2\,\sin(x)\,\cos(x)$

(b) $f''(x) = \big(-\sin(x) + 2\,\sin(x)\,\cos(x)\big)' = -\big(\sin(x)\big)' + 2\,\big(\sin(x)\,\cos(x)\big)'$
$= -\cos(x) + 2\,\Big(\big(\sin(x)\big)'\cos(x) + \sin(x)\,\big(\cos(x)\big)'\Big)$
$= -\cos(x) + 2\,\Big(\cos(x)\,\cos(x) + \sin(x)\big(-\sin(x)\big)\Big)$
$= -\cos(x) + 2\,\cos^2(x) - 2\,\sin^2(x)$

5) (a) $$f'(x) = \left(\frac{4\,\cos^2(x) - 1}{\cos(x)}\right)'$$
$$= \frac{\big(4\,\cos^2(x) - 1\big)'\,\cos(x) - \big(4\,\cos^2(x) - 1\big)\,\cos'(x)}{\cos^2(x)}$$
$$= \frac{8\,\cos(x)\,\cos'(x)\,\cos(x) - \big(4\,\cos^2(x) - 1\big)\,\big(-\sin(x)\big)}{\cos^2(x)}$$
$$= \frac{-8\,\cos^2(x)\,\sin(x) + 4\,\cos^2(x)\,\sin(x) - \sin(x)}{\cos^2(x)}$$
$$= \frac{-4\,\cos^2(x)\,\sin(x) - \sin(x)}{\cos^2(x)} = -\frac{\sin(x)\,\big(4\,\cos^2(x) + 1\big)}{\cos^2(x)}$$

(b) $$f''(x) = \left(-\frac{\sin(x)\,\big(4\,\cos^2(x) + 1\big)}{\cos^2(x)}\right)'$$
$$= \frac{-\Big(\sin(x)\,\big(4\,\cos^2(x) + 1\big)\Big)'\,\cos^2(x) + \sin(x)\,\big(4\,\cos^2(x) + 1\big)\,\big(\cos^2(x)\big)'}{\cos^4(x)}$$

$$= \frac{-\Big(\big(\sin(x)\big)'\,\big(4\,\cos^2(x)+1\big)+\sin(x)\,\big(4\,\cos^2(x)+1\big)'\Big)\,\cos^2(x)}{\cos^4(x)}$$

$$+\,\frac{\sin(x)\,\big(4\,\cos^2(x)+1\big)\,2\,\cos(x)\,\cos'(x)}{\cos^4(x)}$$

$$= \frac{-\Big(\cos(x)\,\big(4\,\cos^2(x)+1\big)+\sin(x)\,8\,\cos(x)\,\overbrace{\cos'(x)}^{-\sin(x)}\Big)\,\cos^2(x)}{\cos^4(x)}$$

$$+\,\frac{\sin(x)\,\big(4\,\cos^2(x)+1\big)\,2\,\cos(x)\,\big(-\sin(x)\big)}{\cos^4(x)}$$

$$= \frac{-4\,\cos^5(x)-\cos^3(x)+8\,\cos^3(x)\,\sin^2(x)}{\cos^4(x)}$$

$$-\,\frac{8\,\cos^3(x)\,\sin^2(x)+2\,\cos(x)\,\sin^2(x)}{\cos^4(x)}$$

$$= -\frac{4\,\cos^5(x)+\cos^3(x)+2\,\cos(x)\,\sin^2(x)}{\cos^4(x)}$$

$$= -\frac{\cos(x)\,\big(4\,\cos^4(x)+\cos^2(x)+2\,\sin^2(x)\big)}{\cos^4(x)}$$

$$= -\frac{4\,\cos^4(x)+\cos^2(x)+2\,\sin^2(x)}{\cos^3(x)}$$

6) (a) $f'(x) = \big(3\,\tan^2(x)-4\,\sqrt{3}\,\tan(x)+3\big)'$

$$= 6\,\tan(x)\,\tan'(x)-4\,\sqrt{3}\,\big(1+\tan^2(x)\big)$$

$$= 6\,\tan(x)\,\big(1+\tan^2(x)\big)-4\,\sqrt{3}\,\big(1+\tan^2(x)\big)$$

$$= 2\,\big(1+\tan^2(x)\big)\,\big(3\,\tan(x)-2\,\sqrt{3}\big)$$

(b) $f''(x) = \Big(2\,\big(1+\tan^2(x)\big)\,\big(3\,\tan(x)-2\,\sqrt{3}\big)\Big)'$

$$= 2\,\Big(\big(1+\tan^2(x)\big)'\,\big(3\,\tan(x)-2\,\sqrt{3}\big)+\big(1+\tan^2(x)\big)\,\big(3\,\tan(x)-2\,\sqrt{3}\big)'\Big)$$

$$= 2\,\Big(2\,\tan(x)\,\tan'(x)\,\big(3\,\tan(x)-2\,\sqrt{3}\big)+\big(1+\tan^2(x)\big)\,3\,\tan'(x)\Big)$$

$$= 2\,\tan'(x)\,\Big(2\,\tan(x)\,\big(3\,\tan(x)-2\,\sqrt{3}\big)+3\,\big(1+\tan^2(x)\big)\Big)$$

$$= 2\,\big(1+\tan^2(x)\big)\,\big(6\,\tan^2(x)-4\,\sqrt{3}\,\tan(x)+3+3\,\tan^2(x)\big)$$

$$= 2\,\big(1+\tan^2(x)\big)\,\big(9\,\tan^2(x)-4\,\sqrt{3}\,\tan(x)+3\big)$$