8.12 Il suffit de vérifier que $f'(x) = 0$.

$$\begin{aligned}
f'(x) &= \big(\sin^6(x) + \cos^6(x) + 3\,\sin^2(x)\,\cos^2(x)\big)' \\
&= 6\,\sin^5(x)\,\sin'(x) + 6\,\cos^5(x)\,\cos'(x) + 3\Big(\big(\sin^2(x)\big)'\cos^2(x) + \sin^2(x)\,\big(\cos^2(x)\big)'\Big) \\
&= 6\,\sin^5(x)\,\cos(x) - 6\,\cos^5(x)\,\sin(x) \\
&\quad + 3\,\big(2\,\sin(x)\,\sin'(x)\,\cos^2(x) + \sin^2(x)\,2\,\cos(x)\,\cos'(x)\big) \\
&= 6\,\sin^5(x)\,\cos(x) - 6\,\sin(x)\,\cos^5(x) + 6\,\sin(x)\,\cos^3(x) - 6\,\sin^3(x)\,\cos(x) \\
&= 6\,\sin(x)\,\cos(x)\,\big(\sin^4(x) - \cos^4(x) + \cos^2(x) - \sin^2(x)\big) \\
&= 6\,\sin(x)\,\cos(x)\,\big(\sin^4(x) - \sin^2(x) - \cos^4(x) + \cos^2(x)\big) \\
&= 6\,\sin(x)\,\cos(x)\,\Big(\sin^2(x)\,\big(\sin^2(x) - 1\big) - \cos^2(x)\,\big(\cos^2(x) - 1\big)\Big) \\
&= 6\,\sin(x)\,\cos(x)\,\Big(\sin^2(x)\,\big(-\cos^2(x)\big) - \cos^2(x)\,\big(-\sin^2(x)\big)\Big) \\
&= 6\,\sin(x)\,\cos(x)\,\big(-\sin^2(x)\,\cos^2(x) + \sin^2(x)\,\cos^2(x)\big) \\
&= 6\,\sin(x)\,\cos(x)\cdot 0 \\
&= 0
\end{aligned}$$