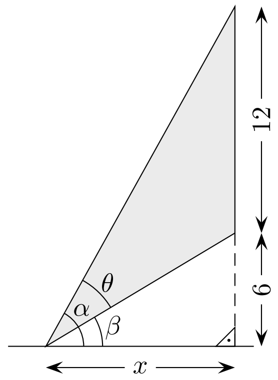
8.24 1) (a) $\tan(\alpha) = \dfrac{6+12}{x} = \dfrac{18}{x}$

$\alpha = \arctan\left(\dfrac{18}{x}\right)$

(b) $\tan(\beta) = \dfrac{6}{x}$

$\beta = \arctan\left(\dfrac{6}{x}\right)$



2) $\theta = \alpha - \beta = \arctan\left(\dfrac{18}{x}\right) - \arctan\left(\dfrac{6}{x}\right) = \theta(x)$

3) $$\theta'(x) = \left(\arctan\left(\frac{18}{x}\right) - \arctan\left(\frac{6}{x}\right)\right)'$$

$$= \arctan'\left(\frac{18}{x}\right)\left(\frac{18}{x}\right)' - \arctan'\left(\frac{6}{x}\right)\left(\frac{6}{x}\right)'$$

$$= \frac{1}{1+\left(\frac{18}{x}\right)^2}\left(-\frac{18}{x^2}\right) - \frac{1}{1+\left(\frac{6}{x}\right)^2}\left(-\frac{6}{x^2}\right)$$

$$= -\frac{\frac{18}{x^2}}{1+\frac{324}{x^2}} + \frac{\frac{6}{x^2}}{1+\frac{36}{x^2}} = -\frac{\frac{18}{x^2}}{\frac{x^2+324}{x^2}} + \frac{\frac{6}{x^2}}{\frac{x^2+36}{x^2}}$$

$$= -\frac{18}{x^2+324} + \frac{6}{x^2+36} = \frac{-18\,(x^2+36) + 6\,(x^2+324)}{(x^2+324)\,(x^2+36)}$$

$$= \frac{-18\,x^2 - 648 + 6\,x^2 + 1944}{(x^2+324)\,(x^2+36)} = \frac{1296 - 12\,x^2}{(x^2+324)\,(x^2+36)}$$

$$= \frac{12\,(108 - x^2)}{(x^2+324)\,(x^2+36)} = \frac{12\,(6\sqrt{3}+x)\,(6\sqrt{3}-x)}{(x^2+324)\,(x^2+36)}$$

		$-6\sqrt{3}$		$6\sqrt{3}$	
12	$+$		$+$		$+$
$6\sqrt{3}+x$	$-$	0	$+$		$+$
$6\sqrt{3}-x$	$+$		$+$	0	$-$
x^2+324	$+$		$+$		$+$
x^2+36	$+$		$+$		$+$
θ'	$-$	0	$+$	0	$-$
θ	$\searrow$	min	$\nearrow$	max	$\searrow$

L'angle θ est ainsi maximal si $x = 6\sqrt{3} \approx 10{,}39$.