3.13 1) (a) $\lim\limits_{x\to-\infty} \sqrt{x^2+x-1} - x = +\infty - (-\infty) = +\infty + \infty = +\infty$

(b) $\lim\limits_{x\to+\infty} \sqrt{x^2+x-1} - x = +\infty - (+\infty) = +\infty - \infty$: indéterminé

$$\lim_{x\to+\infty} \sqrt{x^2+x-1} - x = \lim_{x\to+\infty} \frac{(\sqrt{x^2+x-1}-x)\,(\sqrt{x^2+x-1}+x)}{\sqrt{x^2+x-1}+x} =$$

$$\lim_{x\to+\infty} \frac{(x^2+x-1)-x^2}{\sqrt{x^2+x-1}+x} = \lim_{x\to+\infty} \frac{x-1}{\sqrt{x^2+x-1}+x} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2}+x} =$$

$$\lim_{x\to+\infty} \frac{x}{|x|+x} = \lim_{x\to+\infty} \frac{x}{x+x} = \lim_{x\to+\infty} \frac{x}{2\,x} = \lim_{x\to+\infty} \frac{1}{2} = \frac{1}{2}$$

2) (a) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2-4\,x+3}}{x+1} = \dfrac{+\infty}{-\infty}$: indéterminé

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-4\,x+3}}{x+1} = \lim_{x\to-\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to-\infty} \frac{|x|}{x} = \lim_{x\to-\infty} \frac{-x}{x} =$$
$$\lim_{x\to-\infty} -1 = -1$$

(b) $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2-4\,x+3}}{x+1} = \dfrac{+\infty}{+\infty}$: indéterminé

$$\lim_{x\to+\infty} \frac{\sqrt{x^2-4\,x+3}}{x+1} = \lim_{x\to+\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to+\infty} \frac{|x|}{x} = \lim_{x\to+\infty} \frac{x}{x} =$$
$$\lim_{x\to+\infty} 1 = 1$$

3) (a) $\lim\limits_{x\to-\infty} 5\,x + 3\,\sqrt{x^2+1} = -\infty + \infty$: indéterminé

$$\lim_{x\to-\infty} 5\,x + 3\,\sqrt{x^2+1} = \lim_{x\to-\infty} \frac{(5\,x+3\,\sqrt{x^2+1})\,(5\,x-3\,\sqrt{x^2+1})}{5\,x-3\,\sqrt{x^2+1}} =$$

$$\lim_{x\to-\infty} \frac{(5\,x)^2-3^2(x^2+1)}{5\,x-3\,\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{16\,x^2-9}{5\,x-3\,\sqrt{x^2}} = \lim_{x\to-\infty} \frac{16\,x^2}{5\,x-3\,|x|} =$$

$$\lim_{x\to-\infty} \frac{16\,x^2}{5\,x+3\,x} = \lim_{x\to-\infty} \frac{16\,x^2}{8\,x} = \lim_{x\to-\infty} 2\,x = -\infty$$

(b) $\lim\limits_{x\to+\infty} 5\,x + 3\,\sqrt{x^2+1} = +\infty + \infty = +\infty$

4) (a) $\lim\limits_{x\to-\infty} x - \sqrt{x^2+1} = -\infty - (+\infty) = -\infty - \infty = -\infty$

(b) $\lim\limits_{x\to+\infty} x - \sqrt{x^2+1} = +\infty - (+\infty) = +\infty - \infty$: indéterminé

$$\lim_{x\to+\infty} x - \sqrt{x^2+1} = \lim_{x\to+\infty} \frac{(x-\sqrt{x^2+1})\,(x+\sqrt{x^2+1})}{x+\sqrt{x^2+1}} =$$

$$\lim_{x\to+\infty} \frac{x^2-(x^2+1)}{x+\sqrt{x^2+1}} = \lim_{x\to+\infty} \frac{1}{x+\sqrt{x^2}} = \lim_{x\to+\infty} \frac{1}{x+|x|} =$$

$$\lim_{x\to+\infty} \frac{1}{x+x} = \lim_{x\to+\infty} \frac{1}{2\,x} = 0$$

5) (a) $\lim\limits_{x \to -\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1} = +\infty - (+\infty) = +\infty - \infty$: indéterminé

$\lim\limits_{x \to -\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1} =$

$\lim\limits_{x \to -\infty} \dfrac{(\sqrt{x^2+x+1} - \sqrt{x^2+1})\,(\sqrt{x^2+x+1} + \sqrt{x^2+1})}{\sqrt{x^2+x+1} + \sqrt{x^2+1}} =$

$\lim\limits_{x \to -\infty} \dfrac{(x^2+x+1) - (x^2+1)}{\sqrt{x^2+x+1} + \sqrt{x^2+1}} = \lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2} + \sqrt{x^2}} = \lim\limits_{x \to -\infty} \dfrac{x}{|x| + |x|} =$

$\lim\limits_{x \to -\infty} \dfrac{x}{-x-x} = \lim\limits_{x \to -\infty} \dfrac{x}{-2\,x} = \lim\limits_{x \to -\infty} -\dfrac{1}{2} = -\dfrac{1}{2}$

(b) $\lim\limits_{x \to +\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1} = +\infty - (+\infty) = +\infty - \infty$: indéterminé

$\lim\limits_{x \to +\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1} =$

$\lim\limits_{x \to +\infty} \dfrac{(\sqrt{x^2+x+1} - \sqrt{x^2+1})\,(\sqrt{x^2+x+1} + \sqrt{x^2+1})}{\sqrt{x^2+x+1} + \sqrt{x^2+1}} =$

$\lim\limits_{x \to +\infty} \dfrac{(x^2+x+1) - (x^2+1)}{\sqrt{x^2+x+1} + \sqrt{x^2+1}} = \lim\limits_{x \to +\infty} \dfrac{x}{\sqrt{x^2} + \sqrt{x^2}} = \lim\limits_{x \to +\infty} \dfrac{x}{|x| + |x|} =$

$\lim\limits_{x \to +\infty} \dfrac{x}{x+x} = \lim\limits_{x \to +\infty} \dfrac{x}{2\,x} = \lim\limits_{x \to +\infty} \dfrac{1}{2} = \dfrac{1}{2}$

6) (a) $\lim\limits_{x \to -\infty} \sqrt{x^2+2\,x} - \sqrt{x^2+4} = +\infty - (+\infty) = +\infty - \infty$: indéterminé

$\lim\limits_{x \to -\infty} \sqrt{x^2+2\,x} - \sqrt{x^2+4} =$

$\lim\limits_{x \to -\infty} \dfrac{(\sqrt{x^2+2\,x} - \sqrt{x^2+4})\,(\sqrt{x^2+2\,x} + \sqrt{x^2+4})}{\sqrt{x^2+2\,x} + \sqrt{x^2+4}} =$

$\lim\limits_{x \to -\infty} \dfrac{(x^2+2\,x) - (x^2+4)}{\sqrt{x^2+2\,x} + \sqrt{x^2+4}} = \lim\limits_{x \to -\infty} \dfrac{2\,x-4}{\sqrt{x^2} + \sqrt{x^2}} = \lim\limits_{x \to -\infty} \dfrac{2\,x}{|x| + |x|} =$

$\lim\limits_{x \to -\infty} \dfrac{2\,x}{-x-x} = \lim\limits_{x \to -\infty} \dfrac{2\,x}{-2\,x} = \lim\limits_{x \to -\infty} -1 = -1$

(b) $\lim\limits_{x \to +\infty} \sqrt{x^2+2\,x} - \sqrt{x^2+4} = +\infty - (+\infty) = +\infty - \infty$: indéterminé

$\lim\limits_{x \to +\infty} \sqrt{x^2+2\,x} - \sqrt{x^2+4} =$

$\lim\limits_{x \to +\infty} \dfrac{(\sqrt{x^2+2\,x} - \sqrt{x^2+4})\,(\sqrt{x^2+2\,x} + \sqrt{x^2+4})}{\sqrt{x^2+2\,x} + \sqrt{x^2+4}} =$

$\lim\limits_{x \to +\infty} \dfrac{(x^2+2\,x) - (x^2+4)}{\sqrt{x^2+2\,x} + \sqrt{x^2+4}} = \lim\limits_{x \to +\infty} \dfrac{2\,x-4}{\sqrt{x^2} + \sqrt{x^2}} = \lim\limits_{x \to +\infty} \dfrac{2\,x}{|x| + |x|} =$

$\lim\limits_{x \to +\infty} \dfrac{2\,x}{x+x} = \lim\limits_{x \to +\infty} \dfrac{2\,x}{2\,x} = \lim\limits_{x \to +\infty} 1 = 1$

7) (a) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+1}-x}{x+1} = \dfrac{+\infty-(-\infty)}{-\infty} = \dfrac{+\infty}{-\infty}$: indéterminé

$\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+1}-x}{x+1} = \lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2}-x}{x} = \lim\limits_{x\to-\infty} \dfrac{|x|-x}{x} =$

$\lim\limits_{x\to-\infty} \dfrac{-x-x}{x} = \lim\limits_{x\to-\infty} \dfrac{-2\,x}{x} = \lim\limits_{x\to-\infty} -2 = -2$

(b) $\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2+1}-x}{x+1} = \dfrac{+\infty-(+\infty)}{+\infty}$: indéterminé

$\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2+1}-x}{x+1} = \lim\limits_{x\to+\infty} \dfrac{(\sqrt{x^2+1}-x)\,(\sqrt{x^2+1}+x)}{(x+1)\,(\sqrt{x^2+1}+x)} =$

$\lim\limits_{x\to+\infty} \dfrac{(x^2+1)-x^2}{(x+1)\,(\sqrt{x^2+1}+x)} = \lim\limits_{x\to+\infty} \dfrac{1}{x\,(\sqrt{x^2}+x)} = \lim\limits_{x\to+\infty} \dfrac{1}{x\,(|x|+x)} =$

$\lim\limits_{x\to+\infty} \dfrac{1}{x\,(x+x)} = \lim\limits_{x\to+\infty} \dfrac{1}{2\,x^2} = 0$

8) $\lim\limits_{x\to+\infty} \sqrt{x+\sqrt{x}}-\sqrt{x} = +\infty-(+\infty) = +\infty-\infty$: indéterminé

$\lim\limits_{x\to+\infty} \sqrt{x+\sqrt{x}}-\sqrt{x} = \lim\limits_{x\to+\infty} \dfrac{(\sqrt{x+\sqrt{x}}-\sqrt{x})\,(\sqrt{x+\sqrt{x}}+\sqrt{x})}{\sqrt{x+\sqrt{x}}+\sqrt{x}} =$

$\lim\limits_{x\to+\infty} \dfrac{(x+\sqrt{x})-x}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = \lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}} = \lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x\,(1+\frac{1}{\sqrt{x}})}+\sqrt{x}} =$

$\lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{x}} = \lim\limits_{x\to+\infty} \dfrac{\sqrt{x}}{2\,\sqrt{x}} = \lim\limits_{x\to+\infty} \dfrac{1}{2} = \dfrac{1}{2}$