3.3

1) $\lim\limits_{x\to4}\dfrac{\sqrt{x}-2}{x-4}=\dfrac{\sqrt{4}-2}{4-4}=\dfrac{0}{0}$ indéterminé

$\lim\limits_{x\to4}\dfrac{\sqrt{x}-2}{x-4}=\lim\limits_{x\to4}\dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}=\lim\limits_{x\to4}\dfrac{x-4}{(x-4)(\sqrt{x}+2)}=$

$\lim\limits_{x\to4}\dfrac{1}{\sqrt{x}+2}=\dfrac{1}{\sqrt{4}+2}=\dfrac{1}{4}$

2) $\lim\limits_{x\to1}\dfrac{\sqrt{x}-1}{x-1}=\dfrac{\sqrt{1}-1}{1-1}=\dfrac{0}{0}$ indéterminé

$\lim\limits_{x\to1}\dfrac{\sqrt{x}-1}{x-1}=\lim\limits_{x\to1}\dfrac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}=\lim\limits_{x\to1}\dfrac{x-1}{(x-1)(\sqrt{x}+1)}=$

$\lim\limits_{x\to1}\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{\sqrt{1}+1}=\dfrac{1}{2}$

3) $\lim\limits_{x\to1}\dfrac{\sqrt{x^2+x}-\sqrt{2}}{x-1}=\dfrac{\sqrt{1^2+1}-\sqrt{2}}{1-1}=\dfrac{0}{0}$ indéterminé

$\lim\limits_{x\to1}\dfrac{\sqrt{x^2+x}-\sqrt{2}}{x-1}=\lim\limits_{x\to1}\dfrac{(\sqrt{x^2+x}-\sqrt{2})(\sqrt{x^2+x}+\sqrt{2})}{(x-1)(\sqrt{x^2+x}+\sqrt{2})}=$

$\lim\limits_{x\to1}\dfrac{x^2+x-2}{(x-1)(\sqrt{x^2+x}+\sqrt{2})}=\lim\limits_{x\to1}\dfrac{(x-1)(x+2)}{(x-1)(\sqrt{x^2+x}+\sqrt{2})}=$

$\lim\limits_{x\to1}\dfrac{x+2}{\sqrt{x^2+x}+\sqrt{2}}=\dfrac{1+2}{\sqrt{1^2+1}+\sqrt{2}}=\dfrac{3}{2\sqrt{2}}=\dfrac{3\sqrt{2}}{4}$

4) $\lim\limits_{x\to5}\dfrac{x-5}{\sqrt{2x-1}-3}=\dfrac{5-5}{\sqrt{2\cdot5-1}-3}=\dfrac{0}{0}$ indéterminé

$\lim\limits_{x\to5}\dfrac{x-5}{\sqrt{2x-1}-3}=\lim\limits_{x\to5}\dfrac{(x-5)(\sqrt{2x-1}+3)}{(\sqrt{2x-1}-3)(\sqrt{2x-1}+3)}=$

$\lim\limits_{x\to5}\dfrac{(x-5)(\sqrt{2x-1}+3)}{2x-1-9}=\lim\limits_{x\to5}\dfrac{(x-5)(\sqrt{2x-1}+3)}{2(x-5)}=$

$\lim\limits_{x\to5}\dfrac{\sqrt{2x-1}+3}{2}=\dfrac{\sqrt{2\cdot5-1}+3}{2}=\dfrac{6}{2}=3$

5) $\lim\limits_{x\to0}\dfrac{x^2}{\sqrt{x^2+1}-1}=\dfrac{0^2}{\sqrt{0^2+1}-1}=\dfrac{0}{0}$ indéterminé

$\lim\limits_{x\to0}\dfrac{x^2}{\sqrt{x^2+1}-1}=\lim\limits_{x\to0}\dfrac{x^2(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)}=$

$\lim\limits_{x\to0}\dfrac{x^2(\sqrt{x^2+1}+1)}{x^2+1-1}=\lim\limits_{x\to0}\dfrac{x^2(\sqrt{x^2+1}+1)}{x^2}=\lim\limits_{x\to0}\sqrt{x^2+1}+1=$

$\sqrt{0^2+1}+1=2$

6) $\lim\limits_{x\to2} \dfrac{\sqrt{x+2}-2}{x-2} = \dfrac{\sqrt{2+2}-2}{2-2} = \dfrac{0}{0}$ indéterminé

$$\lim_{x\to2} \frac{\sqrt{x+2}-2}{x-2} = \lim_{x\to2} \frac{(\sqrt{x+2}-2)\,(\sqrt{x+2}+2)}{(x-2)\,(\sqrt{x+2}+2)} =$$

$$\lim_{x\to2} \frac{x+2-4}{(x-2)\,(\sqrt{x+2}+2)} = \lim_{x\to2} \frac{x-2}{(x-2)\,(\sqrt{x+2}+2)} =$$

$$\lim_{x\to2} \frac{1}{\sqrt{x+2}+2} = \frac{1}{\sqrt{2+2}+2} = \frac{1}{4}$$

7) $\lim\limits_{x\to2} \dfrac{3-\sqrt{x+7}}{x-2} = \dfrac{3-\sqrt{2+7}}{2-2} = \dfrac{0}{0}$ indéterminé

$$\lim_{x\to2} \frac{3-\sqrt{x+7}}{x-2} = \lim_{x\to2} \frac{(3-\sqrt{x+7})\,(3+\sqrt{x+7})}{(x-2)\,(3+\sqrt{x+7})} =$$

$$\lim_{x\to2} \frac{9-(x+7)}{(x-2)\,(3+\sqrt{x+7})} = \lim_{x\to2} \frac{-(x-2)}{(x-2)\,(3+\sqrt{x+7})} =$$

$$\lim_{x\to2} \frac{-1}{3+\sqrt{x+7}} = \frac{-1}{3+\sqrt{2+7}} = -\frac{1}{6}$$

8) $\lim\limits_{x\to1} \dfrac{\sqrt{x+8}-\sqrt{4\,x+5}}{x-1} = \dfrac{\sqrt{1+8}-\sqrt{4\cdot1+5}}{1-1} = \dfrac{0}{0}$ indéterminé

$$\lim_{x\to1} \frac{\sqrt{x+8}-\sqrt{4\,x+5}}{x-1} = \lim_{x\to1} \frac{(\sqrt{x+8}-\sqrt{4\,x+5})\,(\sqrt{x+8}+\sqrt{4\,x+5})}{(x-1)\,(\sqrt{x+8}+\sqrt{4\,x+5})} =$$

$$\lim_{x\to1} \frac{(x+8)-(4\,x+5)}{(x-1)\,(\sqrt{x+8}+\sqrt{4\,x+5})} = \lim_{x\to1} \frac{-3\,(x-1)}{(x-1)\,(\sqrt{x+8}+\sqrt{4\,x+5})} =$$

$$\lim_{x\to1} \frac{-3}{\sqrt{x+8}+\sqrt{4\,x+5}} = \frac{-3}{\sqrt{1+8}+\sqrt{4\cdot1+5}} = -\frac{3}{6} = -\frac{1}{2}$$